## Investigation 1 Additional Practice

## 1. a. 65

b. 17
2. Yes; since 2 and 5 are factors of 10 , any number that has 10 as a factor must also have 5 as a factor.
3. No; for example, the number 35 has 7 as a factor, but since it is an odd number it does not have 2 as a factor. 2 and 7 are both prime factors of 14 ; for a number to be a factor of 2 and 7 , it would also be a factor of 14 .
4. a. 29 and 30 , which have a product of $29 \times 30=870$.
b. These two numbers give the largest product because they are the largest numbers.
c. $1,2,3,5,6,10,15,29,30,58,87,145,174$, 290, 435
One way to find the factors is to test factors below 29 to identify factor pairs. We know that $29 \times 30=870$ so the middle factor pair is $29 \times 30$.
5. Answers will vary, but the following are examples of correct answers:
a. $2 \times 5 \times 15$
b. $4 \times 10 \times 25$
c. $2 \times 3 \times 4$
d. $2 \times 3 \times 11$
6. a. $3,5,2$
b. possible answer: $2,8,14$ and $3,12,21$
c. infinitely many
7. a. $6 \times 4=24 ; 4 \times 6=24 ; 24 \div 6=4$; $24 \div 4=6$
b. $8 \times 12=96 ; 12 \times 8=96 ; 96 \div 12=8$;
$96 \div 8=12 ; 3 \times 32=96 ; 32 \times 3=96$;
$96 \div 32=3 ; 96 \div 3=32$
c. $6 \times 18=108 ; 18 \times 6=108 ; 108 \div 6=$
$18 ; 108 \div 18=6 ; 4 \times 27=108 ; 27 \times$
$4=108 ; 108 \div 4=27 ; 108 \div 27=4$;
$9 \times 12=108 ; 12 \times 9=108 ; 108 \div 9=$ $12 ; 108 \div 12=9$
d. A number is called a factor when it is multiplied by another number to find a product. A number is called a divisor of a number when it divides the dividend evenly to find a quotient.

## Skill: Factors, Multiples, and Primes

1. $1,2,3,4,6,12$
2. $1,3,5,9,15,45$
3. 1,41
4. $1,2,3,6,9,18,27,54$
5. $1,2,3,4,6,8,12,16,24,48$
6. $1,2,4,5,10,20,25,50,100$
7. $1,3,9,13,39,117$
$\begin{array}{lll}\text { 8. no } & \text { 9. yes 10. yes 11. no }\end{array}$
8. yes
9. no
10. no
11. yes
12. prime 17. composite 18. composite
13. prime
14. composite
15. composite
16. prime
17. composite
18. prime
19. prime
20. composite
21. composite
22. $53,59,61,67,71,73$

## Investigation 2 Additional Practice

1. The dimensions are $3 \times 8$. The possible dimensions are $1 \times 24,2 \times 12,3 \times 8$, and $4 \times 6$. Only the $3 \times 8$ rectangle has dimensions with a sum of 11 .
2. The dimensions are $3 \times 16$. The possible dimensions are $1 \times 48,2 \times 24,3 \times 16$, $4 \times 14$, and $6 \times 8$. Only the $3 \times 16$ rectangle has dimensions with a sum that is a prime number.
3. a. $1 \times 56,2 \times 28,4 \times 14,7 \times 8$.
b. $1 \times 42,2 \times 21,3 \times 14,6 \times 7$.
c. $1 \times 31$
d. $1 \times 80,2 \times 40,4 \times 20,5 \times 16,8 \times 10$.
e. $1 \times 75,3 \times 25,5 \times 15$.
f. $1 \times 108,2 \times 54,3 \times 36,4 \times 27,6 \times 18$, $9 \times 12$.
g. $1 \times 225,3 \times 75,5 \times 45,15 \times 15$.
4. Phillip's number is 18 since the factor pairs $1 \times 18,2 \times 9$, and $3 \times 6$ have the required sums and $18<20$.
5. In order to determine the number of tiles in each of the rectangles, multiply the tiles along the length by the tiles on the width.
a. 60
b. 40
c. 40
6. a.

b. 1 is the only number in the intersection since 13 is a prime number.
c. Answers will vary; examples include factors of two different prime numbers (e.g., 5 and 13) or factors of two different relatively prime numbers (e.g., 11 and 14).
7. a.

b. The numbers in the intersection are the multiples of 10 , which is $5 \times 2$. Every number that is a multiple of 10 must have 5 and 2 as factors since $5 \times 2=10$.
c. 75 would be placed with the multiples of 5 since it is a multiple of 5 but not a multiple of 2.90 would be placed in the intersection since it is a multiple of both 5 and 2.
8. The fourth number is even. Since the first three numbers were even, odd, and odd, respectively, the sum of these three will be even. Thus, an even number must be added to this even sum to produce an even number.

## Investigation 3 Additional Practice

1. a. Susan shouldn't have to wait at all. The \#14 bus should arrive at the mall at 10 A.m. and the \#11 bus should leave the mall for the museum at about 10 A.m. (since the \#11 bus runs every 12 minutes, it leaves at the top of every hour).
b. Susan will have to wait 12 minutes because the \#14 bus should arrive at noon.
c. Both buses are at the mall at 9 A.m., 10 A.m., 11 A.M., and noon because the least common multiple of 15 and 12 is 60.
2. a. One rectangle is made with 42 tiles, and the other is made with 56 tiles. These are the only two even multiples of 7 between 40 and 60 .
b. The rectangle with 42 tiles has a length of 6 , and the rectangle with 56 tiles has a length of 8 . These answers are found by finding the other number in the factor pair with 7 for each rectangle.
c. Students' answers will vary. For $42: 1 \times$ $42,2 \times 21$, or $3 \times 14$. For $56: 1 \times 56,2 \times$ 28 , or $4 \times 14$.
3. Jack will only have a conflict one day per month on the $21^{\text {st }}$. The least common multiple of 3 and 7 is 21 . The next common multiple, 42 , is greater than the number of days in a month.
4. a. Since the numbers are prime, they don't have any proper factors other than 1. Therefore, their least common multiple would be their product.
b. Since the numbers are prime, the only factors each number has is 1 and itself. Therefore, the greatest common factor must be 1 .
5. a. LCM: 24 ; GCF: 4
b. LCM: 105; GCF: 1
c. LCM: 187; GCF: 1
d. LCM: 108; GCF: 36
e. Parts (b) and (c); for part (b), the two numbers are relatively prime. For part (c), the two numbers are prime.
6. 

a. 4
b. 5
c. 10
d. 25
e. The greatest common factor of two numbers is one of the two numbers when the smaller number is a factor of the larger number.

Skill: Least Common Multiple

1. 10
2. 6
3. 24
4. 12
5. 40
6. 30
7. 60
8. 24
9. 45
10. 30
11. 18
12. 18
13. 15
14. 20
15. 63
16. 28
17. at 1:30 P.M.

Skill: Greatest Common Factor

1. 4
2. 9
3. 1
4. 17
5. 12
6. 6
7. 5
8. 5
9. 5
10. 25
11. 7
12. 3
13. 8
14. 5
15. 1
16. 5 17. 18

## Investigation 4 Additional

## Practice

1. a. $630=2 \times 3 \times 3 \times 5 \times 7$
b. $144=2 \times 2 \times 2 \times 2 \times 3 \times 3$
c. $1,011=337 \times 3$
d. $133=7 \times 19$
e. 23 is prime.
2. a.

Maze 924

$\xrightarrow{\text { Enter }}$| 2 | 3 | 7 | 2 |
| :--- | :--- | :--- | :---: |
| 6 | 2 | 7 | 11 |
| 5 | 4 | 9 | 10 |$\xrightarrow{\text { Exit }}$

b.

c.
Maze 38,220

$\xrightarrow{\text { Enter }}$| 14 | 39 | 70 | 91 |
| ---: | ---: | ---: | ---: |
| 7 | 2 | 20 | 60 |
| 42 | 15 | 2 | 2 |
| 98 | 26 | 13 | 7 |

d.

Maze 210

$\xrightarrow{\text { Enter }}$| 3 | 10 | 3 | 14 |
| ---: | ---: | ---: | ---: |
|  | 2 | 3 | 5 |
| 35 | 2 | 105 | 2 |
| 7 | 15 | 6 | 3 |$\xrightarrow{\text { Exit }}$

3. a. $\mathrm{GCF}=5, \mathrm{LCM}=525$
b. $\mathrm{GCF}=27, \mathrm{LCM}=81$
c. $\mathrm{GCF}=9, \mathrm{LCM}=252$
4. 105 ; an odd number cannot have a factor of 2 , and 3,5 , and 7 are the only three primes with a product less than 160.
5. 1,800
6. a. Possible answer: 6 and 36
b. Possible answer 12 and 60
c. The least common multiple is the other number in the pair.
7. a. Yes, the only common factor they have is 1 .
b. Yes, the only common factor they have is 1 .
c. No, even numbers always have a factor of 2 .
d. Make sure that all the factors of the second number differ from the first (except for 1 ).

## Skill: Prime Factorization

1. $4 ; 2 ; 2$
2. $5 ; 5 ; 3 ; 5$
3. $4 ; 2 ; 2 ; 3 ; 7$
4. $35 ; 2 ; 3 ; 5 ; 7$
5. $2 \times 29$
6. $2 \times 2 \times 2 \times 3 \times 3$
7. $2 \times 2 \times 2 \times 5$
8. $2 \times 5 \times 31$
9. 1,540
10. 17,017
11. 4 ways; 2 groups of 16,4 groups of 8 , 8 groups of 4,16 groups of 2
12. $2 \cdot 3 \cdot 13$
13. $2 \cdot 3^{2} \cdot 7$
14. $5^{3}$
15. $2 \cdot 3^{2} \cdot 5$
16. $2^{2} \cdot 23$
17. $2^{2} \cdot 3^{2} \cdot 5$
18. 126
19. 105
20. 72
21. 168
22. 3 packages of hot dogs, 2 packages of buns
23. $3 \quad$ 24. 24
24. 7
25. 29
26. 1,700 and 2,550

## Investigation 5 Additional

## Practice

1. Begin with the number 1 , repeatedly multiply by 3 until the result exceeds 1,000 . The numbers would be $3,9,27,81$, 243, 729.
2. a. Alternate multiplying terms by 5 and then 3. In other words, multiply the first term by 5 to get the second term, multiply the second term by 3 to get the third, multiply the third term by 5 to get the fourth, multiply the fourth term by 3 to get the fifth, and so on.
b. 3,375 and 10,125
c. The greatest common factor is 3 , since it is the first term in the sequence and a prime number.
3. a. Student 1
b. Students 1, 2, and 4
$\begin{array}{ll}\text { 4. a. Locker } 1,000 & \text { b. Locker } 988 \\ \text { c. Locker } 960 & \text { d. Locker } 600\end{array}$
c. Locker 960
d. Locker 600
4. The largest prime desert less than 50 is $\{24,25,26,27,28\}$.
5. 

a. $23 \times 5=115$
b. $21 \times 10=210$
7. a. $190=2 \times 5 \times 19$
b. $319=11 \times 19$
c. $225=3 \times 5 \times 17$ d
d. $406=2 \times 7 \times 29$

