# Investigation 4

# Measuring Parallelograms

In this unit, you have developed ways to find the area and perimeter of rectangles and of triangles. In this investigation you will develop ways to find the area and perimeter of parallelograms.

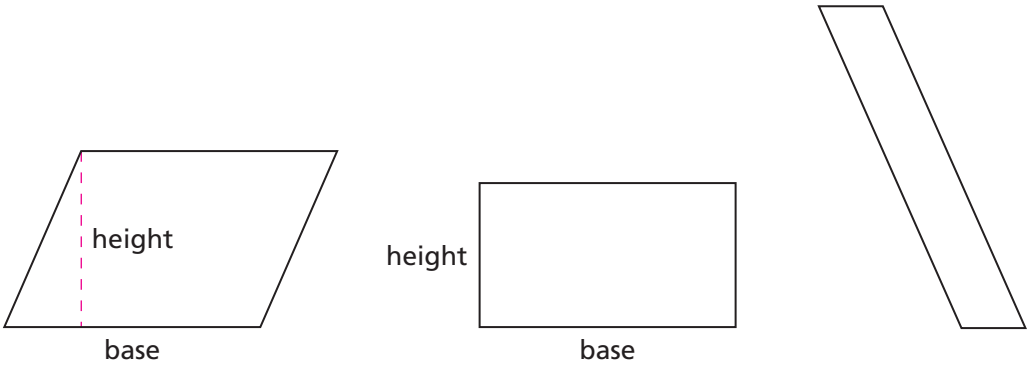
When you work with rectangles, you use measurements like length and width. For triangles, you use the side lengths, the base, and the height. Like triangles, parallelograms are often described by measures of side length, base, and height.

## 4.1 Finding Measures of Parallelograms

As you work with parallelograms, remember what you know about triangles and look for ways to relate these two figures.

## Getting Ready for Problem 4.1

Here are three parallelograms with the base and height of two parallelograms marked. What do you think the *base* and the *height* of a parallelogram mean? How do you mark and measure the base and height of the third figure?




## Problem 4.1 Finding Measures of Parallelograms

Six parallelograms labeled A–F are drawn on the centimeter grid on the next page.

**A. 1.** Find the perimeter of each parallelogram.

**2.** Describe a strategy for finding the perimeter of a parallelogram.

**B. 1.** Find the area of each parallelogram.

**2.** Describe the strategies you used to find the areas.

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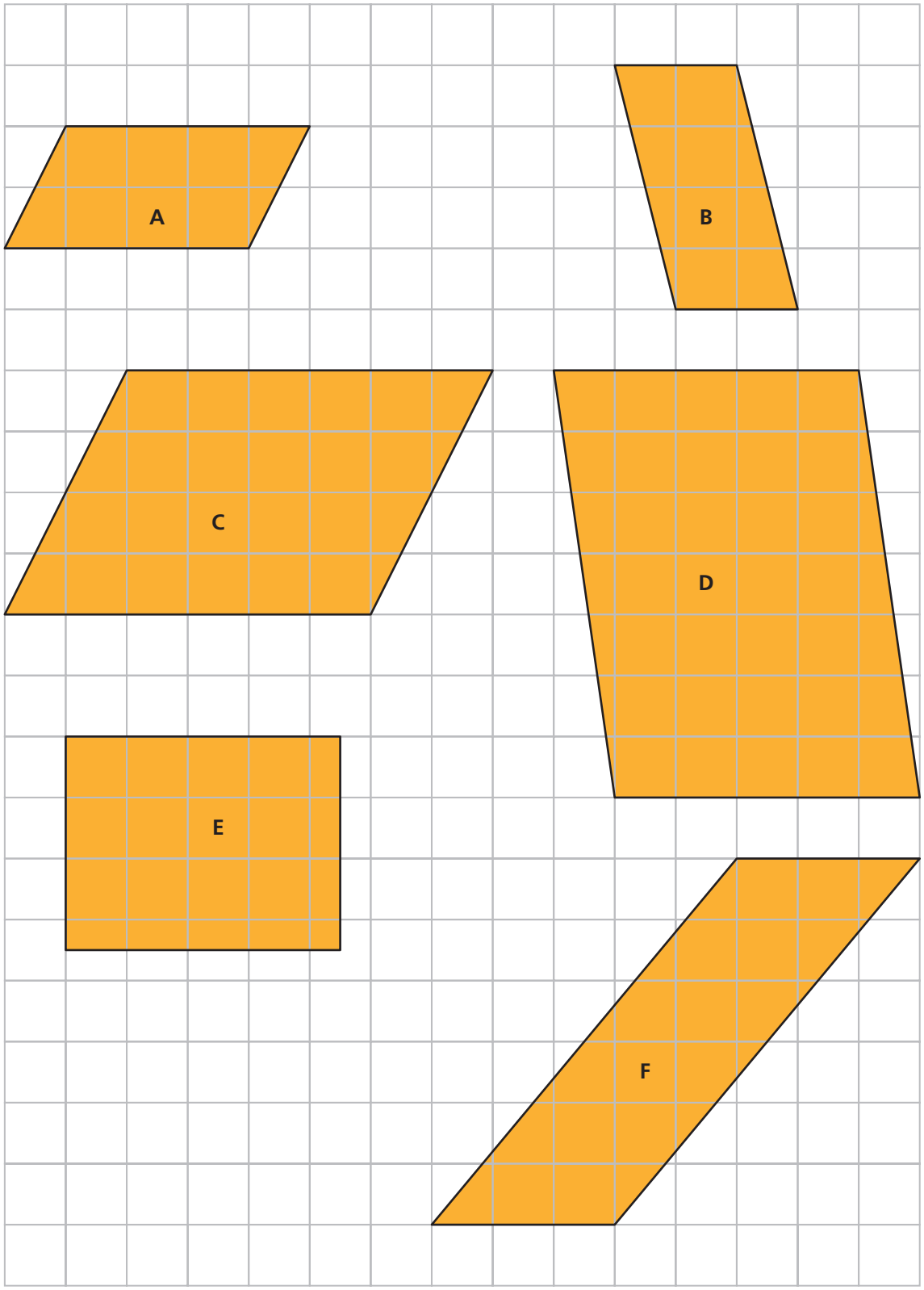

A
B
C
D
E
F

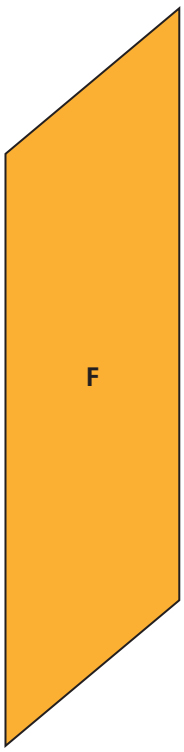
# 4.2 Parallelograms From Triangles

In this problem, you will consider how the area of a parallelogram relates to its base and height. You will also consider how the area of a parallelogram relates to the area of a triangle with the same base and height.

## Problem 4.2 Parallelograms From Triangles

At the right is parallelogram F from Problem 4.1.

Trace two copies of this parallelogram.



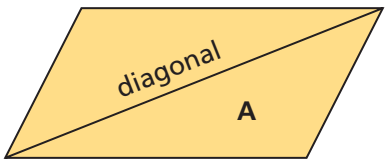

**A. 1.** Find two ways to position parallelogram F on a centimeter grid.

**2.** Record the base and height for each position you find.

**3.** How does the area of the parallelogram relate to the base and height in each position?

**B. 1.** Look at parallelograms A–F from Problem 4.1 again. Make a table recording the area, base, and height of each parallelogram.

**2.** Draw one diagonal in each parallelogram as shown below. Add columns to your table recording the area, base, and height of each triangle.




**3.** Look for patterns in your table that show how the area of each parallelogram and the area of its triangles are related.

**4.** How are the bases and heights of each parallelogram and the triangles made by a diagonal related?

**C. 1.** Write a rule for finding the area of a parallelogram. Use $b$ to represent the base and $h$ to represent the height.

2. Use your rule to find the area of this parallelogram. Make any measurements you need in centimeters.

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## 4.3 Designing Parallelograms Under Constraints

Now you can draw parallelograms that meet given conditions. Sometimes you will be able to draw more than one parallelogram that satisfies the constraints given.

### Problem 4.3 Designing Parallelograms Under Constraints

For each description, draw two figures that are *not* congruent (same shape, same size) to each other. If you can't draw a second figure, explain why. Make your drawings on centimeter grid paper.

**A.** The rectangles each have an area of 18 square centimeters. If you can draw two different rectangles, do they have the same perimeter?

**B.** The rectangles are each 3 centimeters by 8 centimeters. If you can draw two different rectangles, do they have the same area?

**C.** The parallelograms each have a base of 7 centimeters and a height of 4 centimeters. If you can draw two different parallelograms, do they have the same area?

**D.** The parallelograms each have all 6-centimeter side lengths. If you can draw two different parallelograms, do they have the same area?

**E.** The parallelograms each have an area of 30 square centimeters. If you can draw two different parallelograms, do they have the same perimeter?

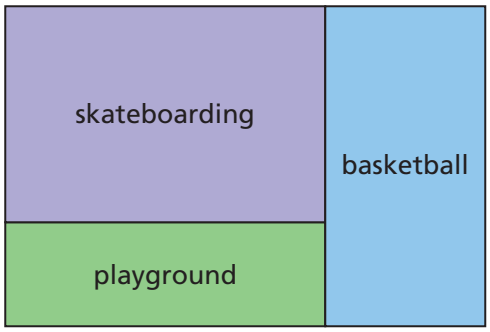
ACE Homework starts on page 60.

# 4.4 Parks, Hotels, and Quilts

Now that you know how to find the area of rectangles, triangles, and parallelograms, here are some problems to test your skills.

## Problem 4.4 Finding Areas and Perimeters

**A.** The Luis Park District set aside a rectangular section of land to make a park. After talking with students, the park district decides to make an area for skateboarding, an area with playground equipment, and an area with a basketball court, as shown.




**1.** A fence surrounds the skateboarding area that takes up $\frac{2}{3}$ of the length and $\frac{2}{3}$ of the width of the park. What fraction of the area of the park does the skateboarding area occupy?

**2.** The basketball court is 35 feet by 60 feet. Use this information and what you know about the skateboarding area to find the area and the perimeter of the playground area.

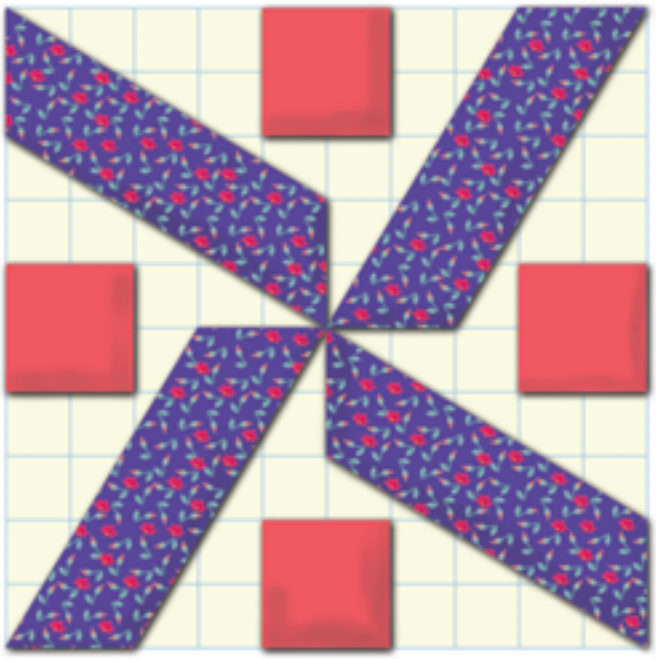
**B.** The Luxor Hotel in Las Vegas is built in the shape of a pyramid. When you look at the pyramid from the outside, each face (side) of the pyramid is a glass equilateral triangle.

**1.** Each face is an equilateral triangle with a base that is 646 feet and a height that is approximately $559\frac{9}{20}$ feet. Sketch a face of the pyramid. Label the base and height.

2. Estimate the area of the glass used to cover one triangular face.

3. If lights are strung along the three edges of one triangular face, how many feet of lights are needed?

**C.** Quilters use shapes such as triangles, squares, rectangles, and parallelograms when designing quilts. This is a pattern of a 10 inch-by-10 inch quilt square on inch grid paper.



= 1 inch

1. Each parallelogram in the quilt is made from how many square inches of fabric?

2. How many square inches of fabric are used to make the small red squares in the quilt square?

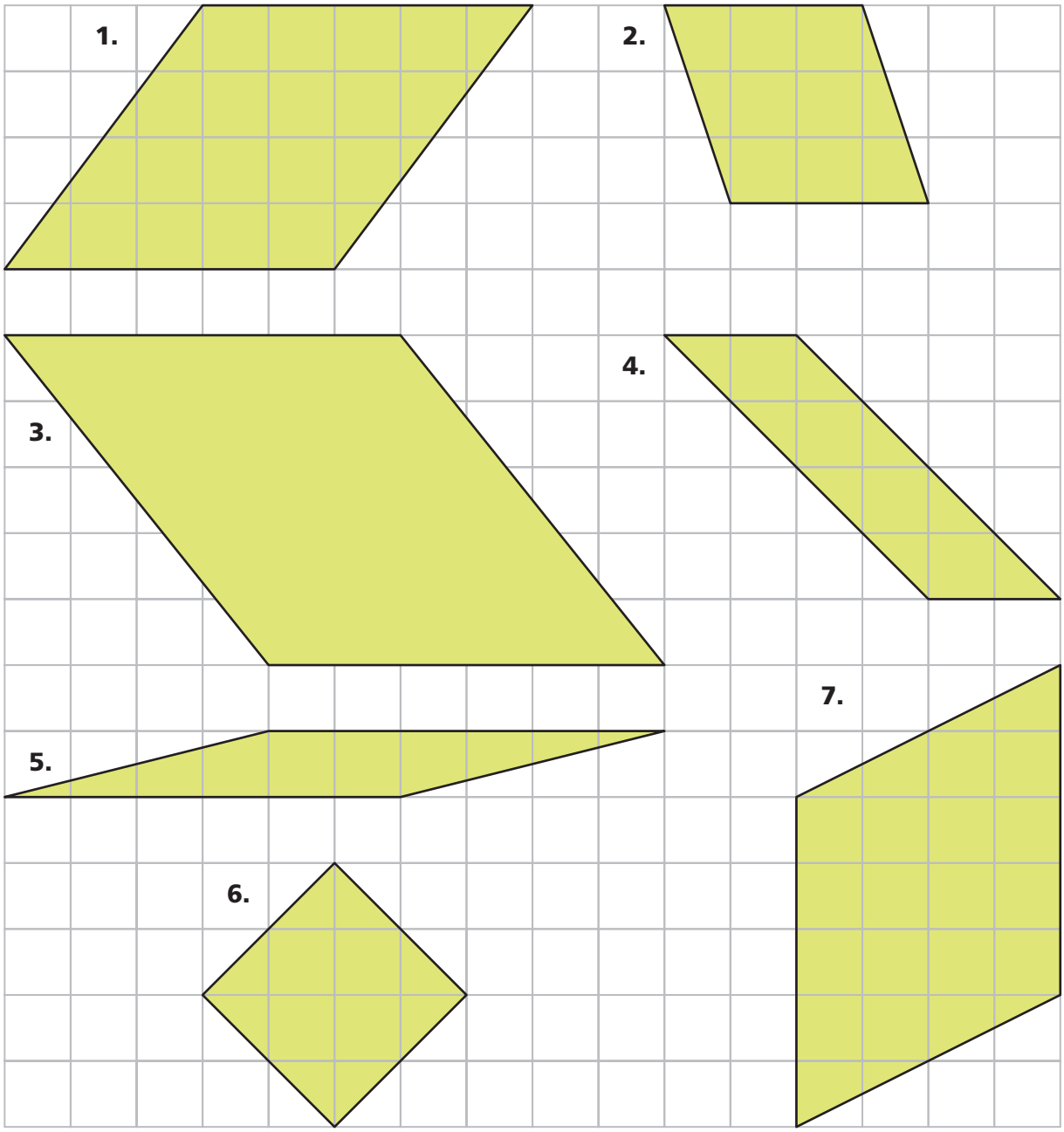
3. The squares and the parallelograms will be sewn onto white fabric. How many square inches of the white fabric will be visible?

**ACE Homework starts on page 60.**

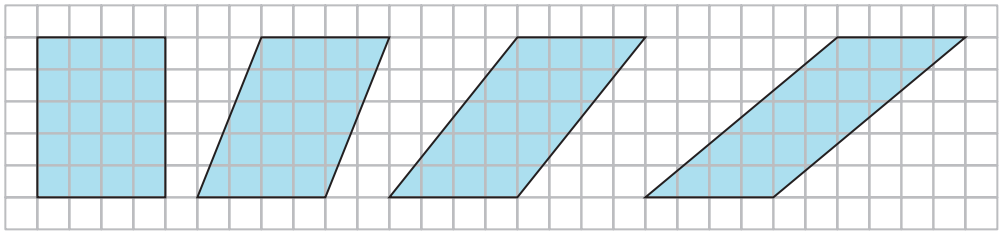
Applications Connections Extensions ACE

## Applications

**For Exercises 1–7, find the area and perimeter of each parallelogram. Give a brief explanation of your reasoning for Exercises 2, 6, and 7.**



**8.** On the grid is a family of parallelograms.



**a.** Find the base, height, and area of each of the parallelograms.

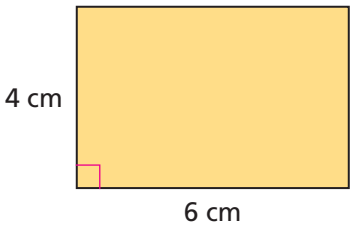
**b.** What patterns do you see?

**c.** Why do you think they are called a family of parallelograms?

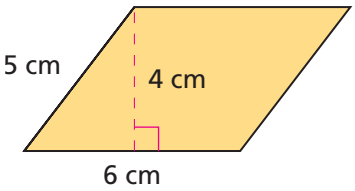
**For Exercises 9–13, find the area and perimeter of each figure. (Figures are not drawn to scale.)**



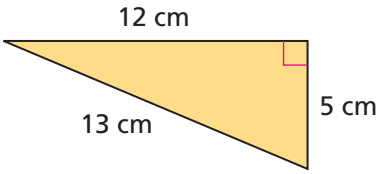
**9.**



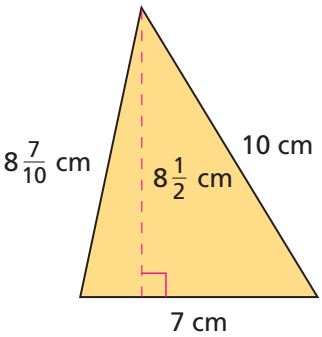

**10.**



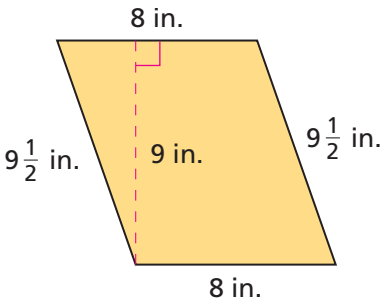

**11.**




**12.**

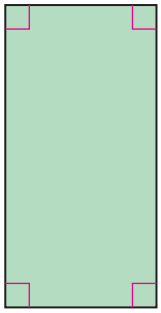



**13.**



**For Exercises 14–19, make the measurements (in centimeters) that you need to find the area and perimeter of each shape. Write your measurements on a sketch of each figure. Then find the area and perimeter of each shape.**

**14.**



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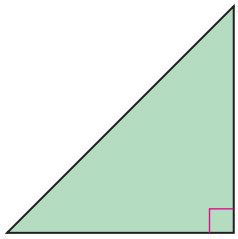
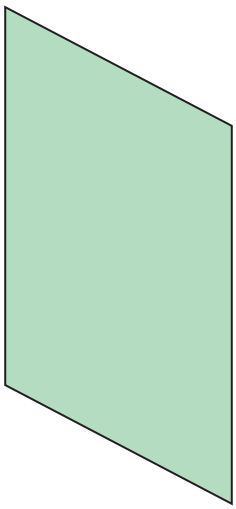
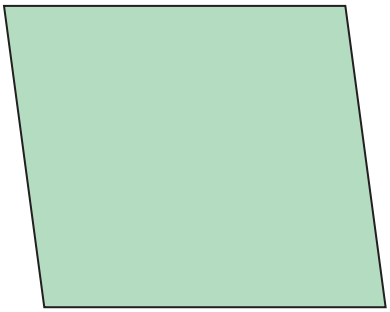


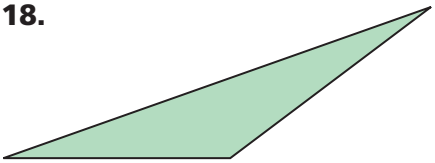
**16.**



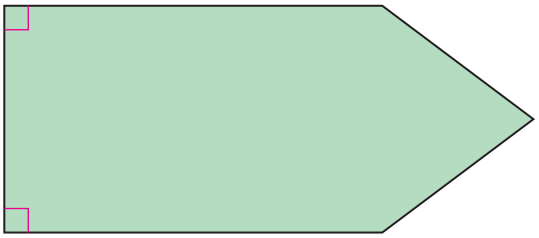
**17.**



**18.**



**19.**

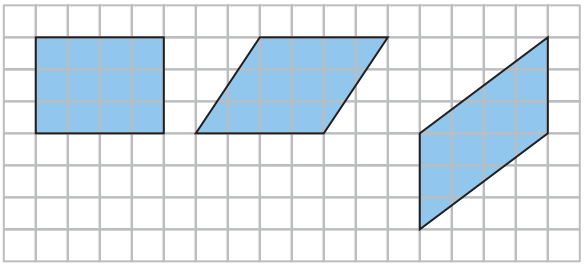


**20.** Denzel decides the shape of Tennessee is approximately that of a parallelogram, as shown below.


**a.** Use the distances shown to estimate the area of Tennessee.

**b.** The actual area of Tennessee is 41,217 square miles. How does your estimate compare to the actual area? Explain.

**21.** Explain why these three parallelograms have the same area.



**For Exercises 22–27:**

**a. Sketch the described parallelogram.**
**b. Label its base and height.**
**c. Explain whether you can draw more than one parallelogram that will meet the given conditions.**

**22.** The base is 8 cm and the perimeter is 28 cm.

**23.** The base is $4\frac{1}{2}$ cm and the area is 27 $cm^2$.

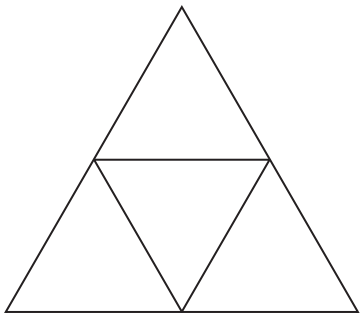
**24.** A non-rectangular parallelogram has a base of 10 cm and a height of 8 cm.

**25.** The base is 6 cm and the area is 30 $cm^2$.

**26.** The area is 24 $cm^2$.

**27.** The perimeter is 24 cm.

**28. a.** An equilateral triangle can be divided into equal-sized triangles using lines parallel to the opposite sides. The lines connect two midpoints. How many parallelograms can you find in the figure?



**b.** Suppose the area of the large triangle is 16 square units. What is the area of each of the parallelograms?

**29.** Akland Middle School plans to make a flowerbed in front of the administration building. The plan involves one main parallelogram surrounded by four small parallelograms as shown.

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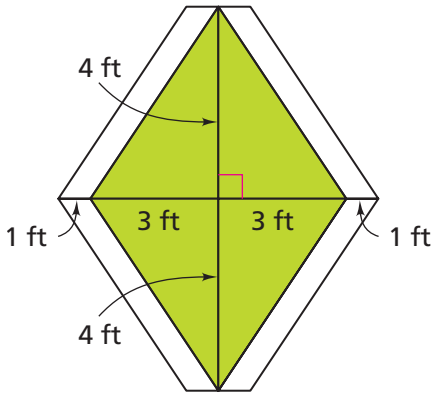



**a.** How many square feet is the area of each of the four small parallelograms?

**b.** How many square feet is the area of the main parallelogram?

**30.** Mr. Lee wants to install ceiling tiles in his recreation room. The room is 24 feet by 18 feet. Each ceiling tile is 2 feet by 3 feet. How many ceiling tiles will he need?

**31.** The Lopez family bought a plot of land in the shape of a parallelogram. It is 100 feet wide (across the front) and 200 feet deep (the height). Their house covers 2,250 square feet of land. How much land is left for grass?

## Connections

**32.** **Multiple Choice** Which set of numbers is ordered from greatest to least?

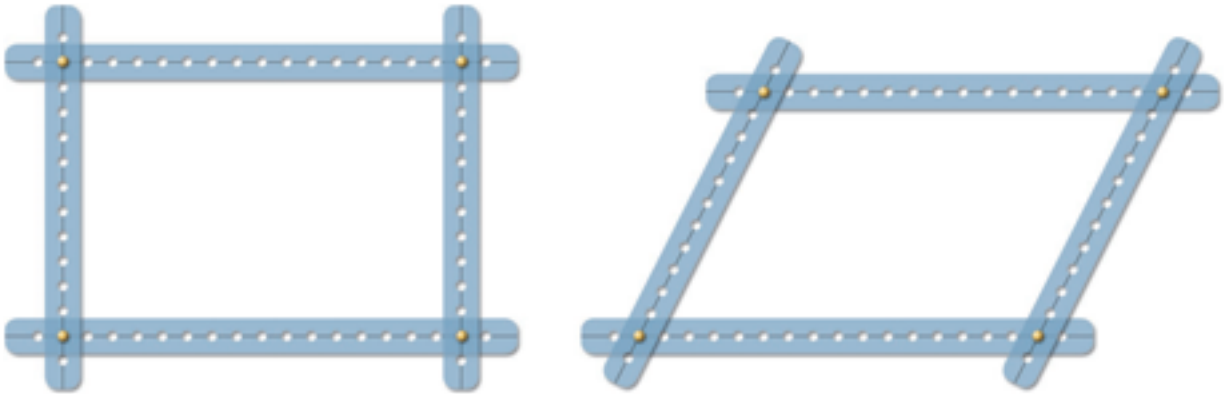
**A.** $0.215, 0.23, 2.3, \frac{2}{3}$

**B.** $\frac{2}{3}, 0.215, 0.23, 2.3$

**C.** $\frac{2}{3}, 0.23, 0.215, 2.3$

**D.** $2.3, \frac{2}{3}, 0.23, 0.215$

**33.** Rectangles made from Polystrips can easily tilt out of shape into another parallelogram.



**a.** Suppose a rectangle made of Polystrips tilts out of shape with the sides staying the same length. How will the angles, area, and perimeter of the new figure compare to the original?

**b.** What relationships among the sides and angles of rectangles are also true of parallelograms?

**34.** **Multiple Choice** Two quadrilaterals are congruent. Which statement is correct?

**F.** They have the same area, but may have different perimeters.

**G.** They have the same perimeters, but may have different areas.

**H.** They may have different perimeters and different areas.

**J.** They have the same area and the same perimeter.

**35.** Give two examples of a pair of congruent quadrilaterals.

**36.** Rapid City is having its annual citywide celebration. The city wants to rent a bumper-car ride. The pieces used to make the floor are 4 foot-by-5 foot rectangles. The ride covers a rectangular space that is 40 feet by 120 feet.

**a.** How many rectangular floor pieces are needed?

**b.** The ride costs $20 per floor piece and $10 per bumper car. How much would it cost Rapid City to rent the floor and the bumper cars? (You will need to decide how many bumper cars will be appropriate.)

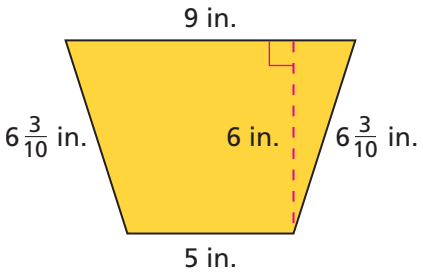
# Extensions

**37.** You saw earlier that in some parallelograms and triangles, the height is outside the shape being measured.

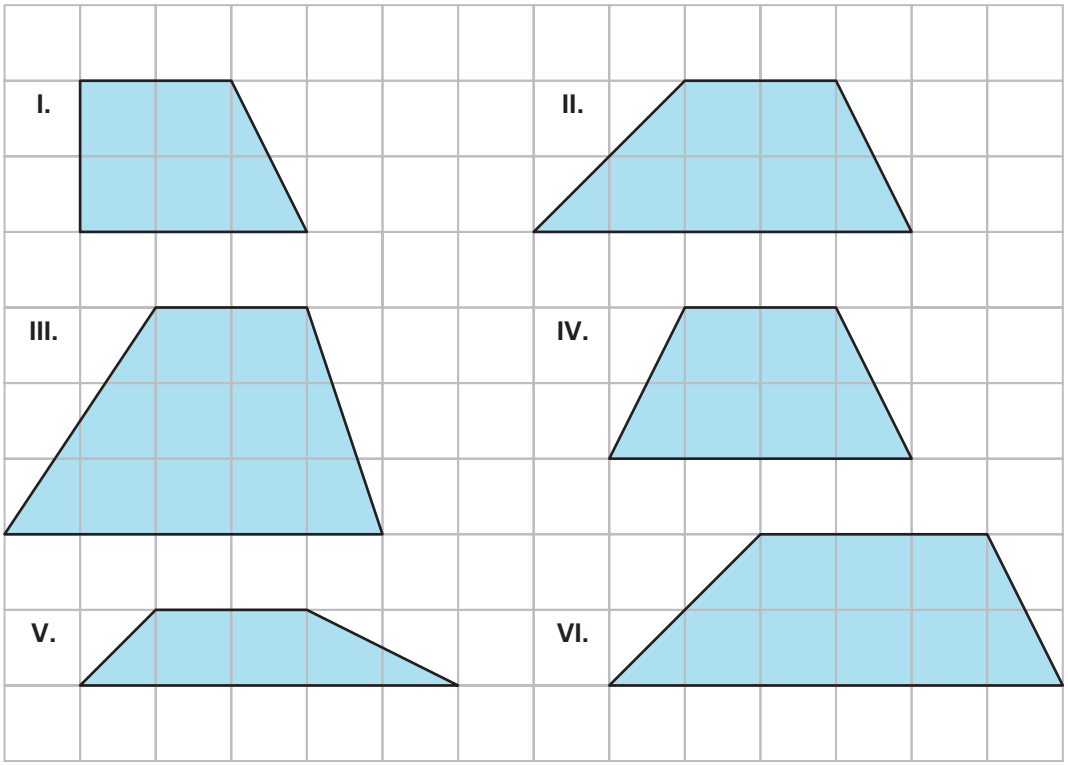
**a.** Sketch an example of a parallelogram with the height outside the parallelogram. Explain why the area of the parallelogram can still be calculated by multiplying the base times the height.

**b.** Sketch an example of a triangle with the height outside the triangle. Explain why the area of the triangle can still be calculated by multiplying $\frac{1}{2}$ times the base times the height.

**38.** Find the area and perimeter of the figure.



**39.** A trapezoid is a polygon with at least two opposite edges parallel. Use these six trapezoids. Make a table to summarize what you find in parts (a) and (c).




**a.** Without counting all the squares, find the area of each trapezoid.

**b.** Summarize your method for part (a) with a rule or a description.

**c.** Find the perimeter of each trapezoid.

**d.** Summarize your method for part (c) with a rule or a description.

# Mathematical Reflections 4

**In this investigation, you developed strategies for finding the area and perimeter of parallelograms. These questions will help you to summarize what you have learned.**

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. Describe an efficient way to find the area of a parallelogram. Include the measurements you would need to make and how you would use them to find the area.
2. How is finding the area of a parallelogram similar to finding the area of a triangle and the area of a rectangle?
3. Describe how to find the perimeter of a parallelogram. Include the measurements you would need to make and how you would use them to find the perimeter.
4. How is finding the perimeter of a parallelogram like finding the perimeter of a triangle and the perimeter of a rectangle?