# Investigation 1

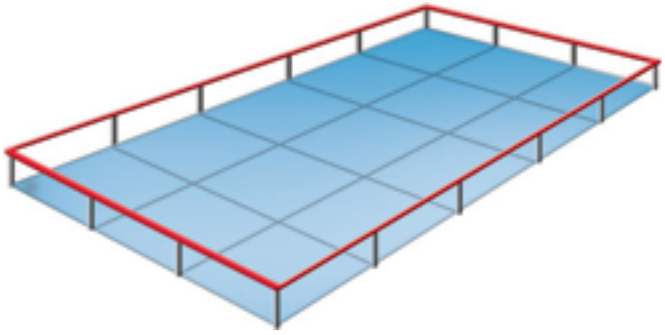
# Designing Bumper Cars

Most people enjoy the rides at amusement parks and carnivals, from merry-go-rounds and Ferris wheels to roller coasters and bumper cars.

Suppose a company called Midway Amusement Rides (MARS for short) builds rides for amusement parks and carnivals. To do well in their business, MARS designers have to use mathematical thinking.

## 1.1 Designing Bumper-Car Rides

Bumper cars are a popular ride at amusement parks and carnivals. Bumper cars ride on a smooth floor with bumper rails all around it. MARS makes their bumper-car floors from 1 meter-by-1 meter square tiles. The bumper rails are built from 1-meter sections.



## Problem 1.1 Understanding Area and Perimeter

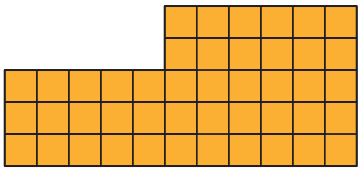
When a customer sends an order, the designers at MARS first use square tiles to model possible floor plans. MARS has received the customer orders below. Experiment with square tiles and then sketch some designs for the customer to consider.

**A.** Badger State Shows in Wisconsin requests a bumper-car ride with 36 square meters of floor space and 26 meters of rail sections. Sketch two or three floor plans for this request.

**B.** Lone Star Carnivals in Texas wants a bumper-car ride that covers 36 square meters of floor space and has lots of rail sections. Sketch two or three possible floor plans for this customer.

**C.** Two measures tell you important facts about the size of the bumper-car floor plans you have designed. The number of tiles needed to cover the floor is the **area.** The number of rail sections needed to surround the floor is the **perimeter.**

**1.** What are the area and perimeter of this bumper-car floor plan?



**2.** Which measure, perimeter or area, do you think better describes the *size* of a bumper-car floor plan? Why?

ACE **Homework starts on page 10.**

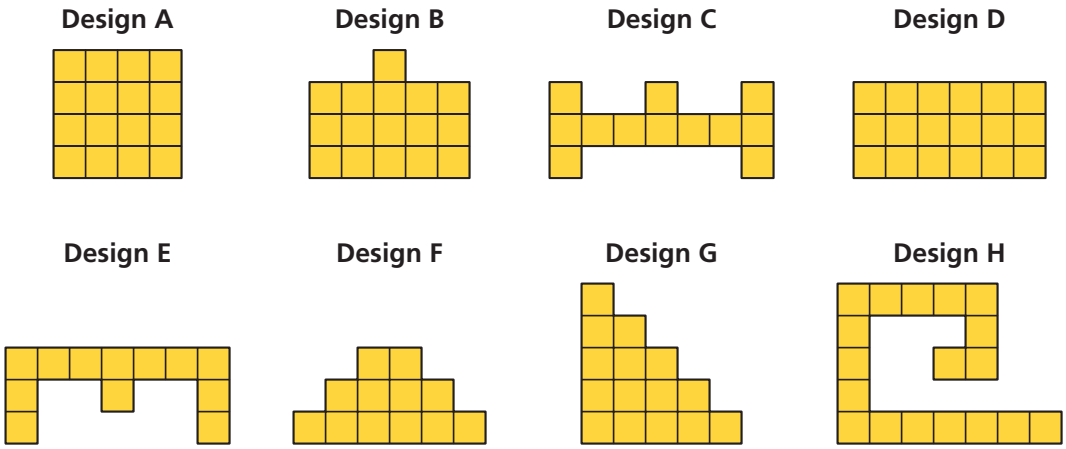
## 1.2 Pricing Bumper-Car Rides

When it is time to prepare the estimates or bills for customers, the designers at MARS turn over the plans to the billing department. The company charges $25 for each rail section and $30 for each floor tile.

## Problem 1.2 Finding Area and Perimeter

The Buckeye Amusement Company in Ohio wants some sample floor plans and cost estimates for bumper-car rides. The designers come up with these bumper-car floor plans.

bumper-car tile: 1 m
1 m




**A.** Find the area and perimeter for each design. Record your data in a table such as the one started at the right.

| Design | Area | Perimeter | Cost |
|---|---|---|---|
| A | | | |
| B | | | |

**B.** Use the data in your table.

1. Which designs can be made from the same number of floor tiles?
2. Choose a set of designs that can be made from the same number of floor tiles. What is the perimeter of each design?
3. In the designs with the same floor area, which design costs the most? Which design costs the least? Why?

**C.** 1. Rearrange the tiles in Design H to form a rectangle. Can you make more than one rectangle using the same number of tiles? If so, are the perimeters of the rectangles the same? Explain.

2. Design B and Design D have the same perimeter. Can you rearrange Design B to make Design D? Explain.

**D.** 1. The Buckeye Amusement Company said that it is willing to pay between $1,000 and $2,000 for a bumper-car ride. Design two possible floor plans. Find the area, perimeter, and cost for each.

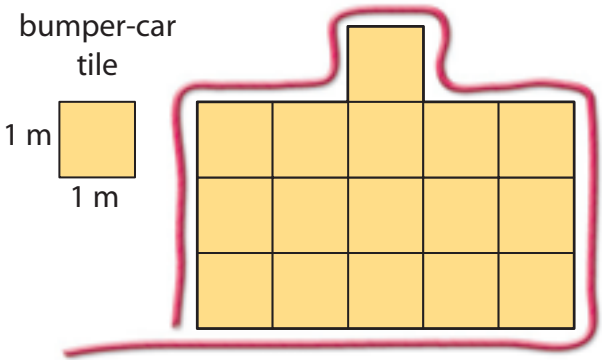
2. Suppose you were the manager. Which design would you choose? Why?

ACE **Homework starts on page 10.**

## Getting Ready for Problem 1.3

A student is tired of counting the individual rail sections around the outside of each bumper-car track. She starts to think of them as one long rail. She wraps a string around the outside of Design B, as shown.

What do you think she does next? How does this help her to find the perimeter of the figure? How could she determine the area?




# 1.3 Decoding Designs

The Portland Community Events Council is planning its annual summer festival. The council asks for bids from different traveling carnival shows. Each carnival show sends descriptions of the rides they offer.

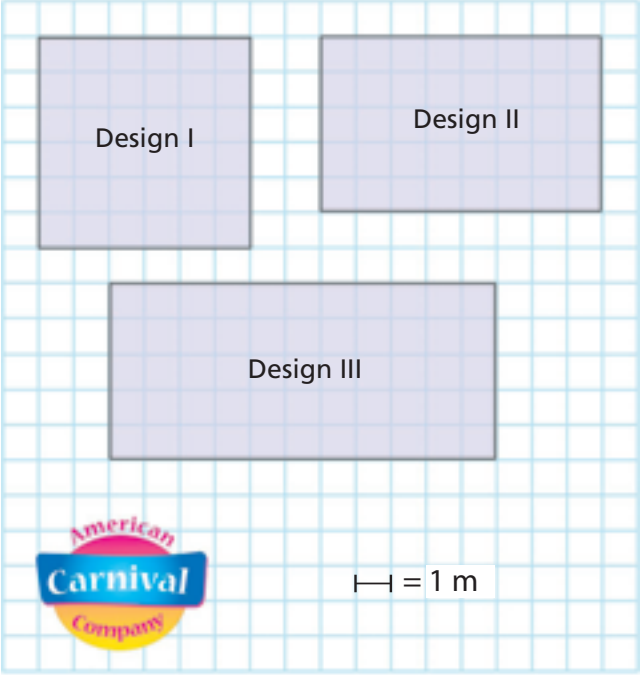
## Problem 1.3 Finding Area and Perimeter of Rectangles

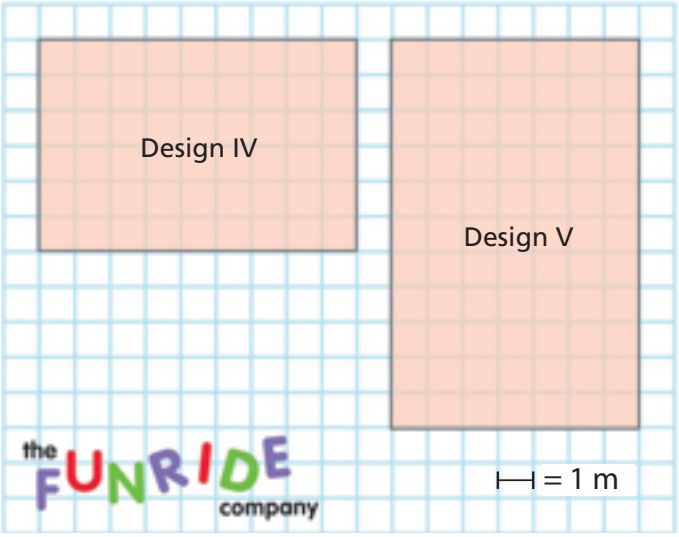
The council wants to have a bumper-car ride in the shape of a rectangle at the festival.

**A.** American Carnival Company sends Designs I, II and III. The Fun Ride Company sends Designs IV and V (on the next page).

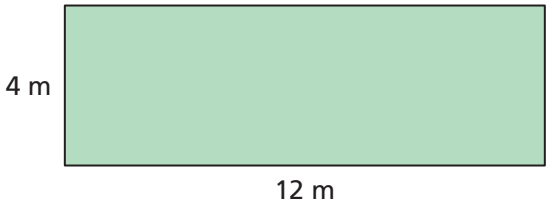
**1.** What is the area of each design? Explain how you found the area.

**2.** What is the perimeter of each design? Explain how you found the perimeter.






**B.** One carnival company sends the rectangular floor plan below. Find the area and the perimeter of this floor plan.



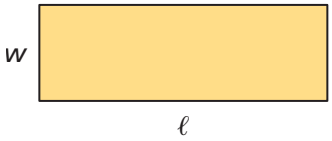

**C.** Another carnival company sends a description rather than a diagram. They describe the ride as a rectangle that is 17 meters by 30 meters.

**1.** What is the area of this floor plan?

**2.** What is the perimeter of this floor plan?

**D.** The dimensions of a rectangle are called **length** and **width.** Length can be represented using $\ell$ and width can be represented using $w$.

**1.** Using $\ell$ for length and $w$ for width, write a rule for finding the perimeter of a rectangle.




**2.** Using $\ell$ for length and $w$ for width, write a rule for finding the area of a rectangle.

ACE **Homework starts on page 10.**

## Applications

1. Coney Island Park wants a bumper-car ride with 24 square meters of floor space and 22 meters of rail section.
   a. Sketch some floor plans for this request.
   b. Describe the bumper-car ride in terms of its area and perimeter. Report what each measure tells you about the ride.

### Did You Know?

Bumper cars came from the Dodgem, a rear-steering car invented by Max and Harold Stoeher of Methuen, Massachusetts. The Dodgem's popularity drew the attention of cousins Joseph and Robler Lusse, who made roller coaster parts in their Philadelphia machine shop.

The Lusses knew that people like to bump into each other, and also want to choose who to bump. So they worked on designs that let a bumper car go from forward to reverse without going through neutral. They filed the first of 11 patents in 1922 for their bumper car.

Go Online PHSchool.com **For:** Information about bumper cars **Web Code:** ame-9031

**For Exercises 2–5, experiment with tiles or square grid paper. Sketch each answer on grid paper.**

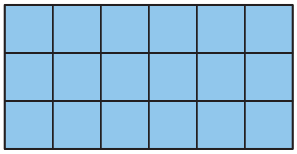
**2.** Draw two different shapes with an area of 16 square units. What is the perimeter of each shape?

**3.** Draw two different shapes with a perimeter of 16 units. What is the area of each shape?

**4.** Draw two different shapes with an area of 6 square units and a perimeter of 12 units.

**5.** Draw two different shapes with an area of 15 square units and a perimeter of 16 units.

**6.** Use this design for parts (a) and (b).

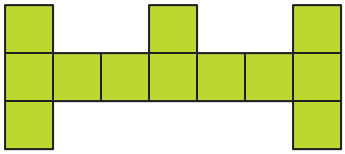


**a.** If possible, draw a figure with the same area, but with a perimeter of 20 units. If this is not possible, explain why.

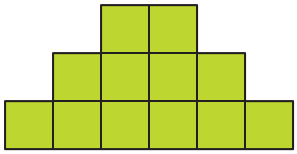
**b.** If possible, draw a figure with the same area, but with a perimeter of 28 units. If this is not possible, explain why.

**7.** These designs have an area of 12 square meters. Are the perimeters the same? Explain how you decided.

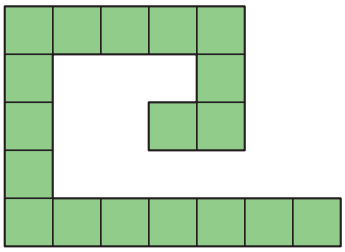
**Design C**



**Design F**



**8.** Copy the design below onto grid paper. Add six squares to make a new design with a perimeter of 30 units. Explain how the perimeter changes as you add tiles to the figure.



**For Exercises 9–12, each unit length represents 12 feet. Find the area and perimeter of each floor plan.**

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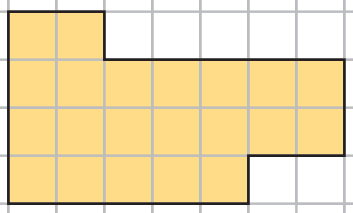
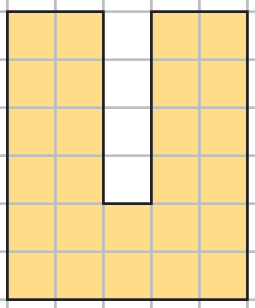
**Web Code:** ama-5154

**9.**

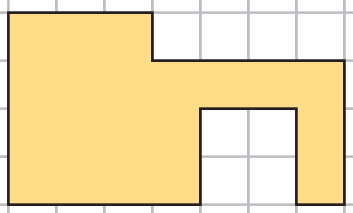
**10.**



**11.**

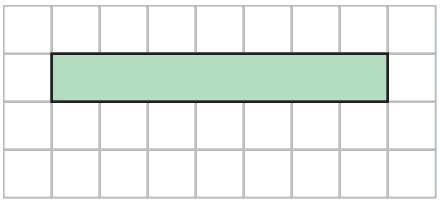


**12.**

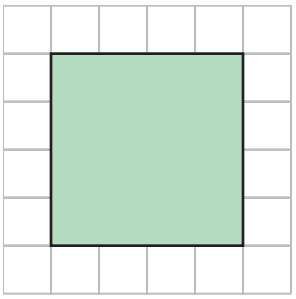


**For Exercises 13–20, find the area and perimeter of each shaded rectangle.**

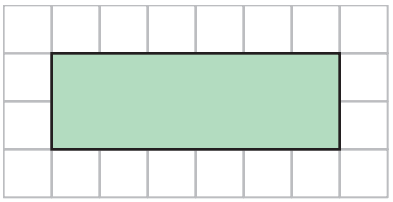
**13.**



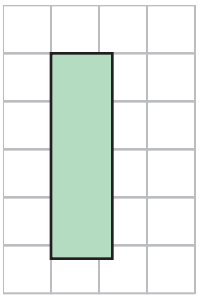
**14.**



**15.**

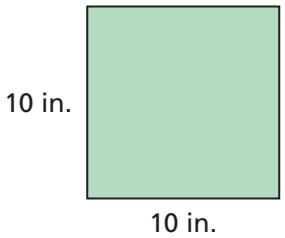


**16.**

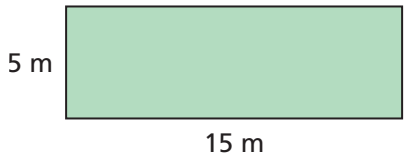


**17.**

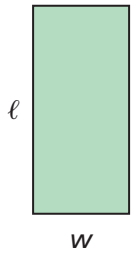

**18.**



**19.**




**20.**



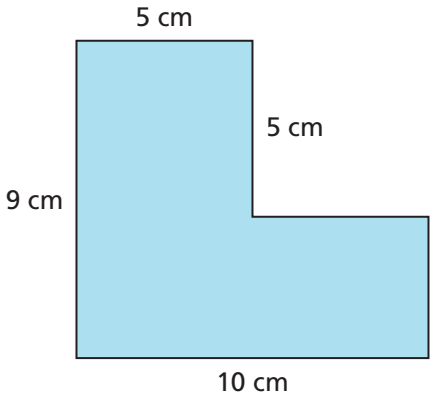

**21.** Copy and complete the table. Sketch each rectangle and label its dimensions.

**Rectangle Area and Perimeter**

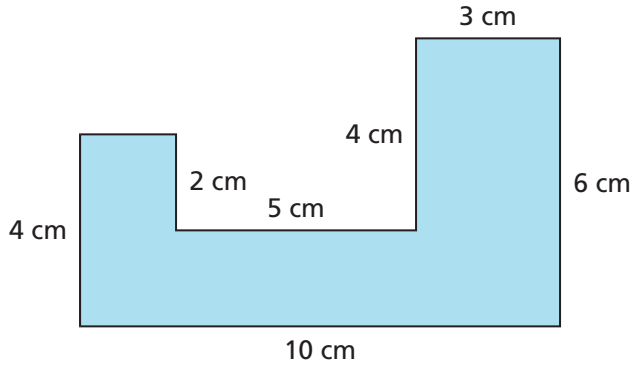
| Rectangle | Length | Width | Area | Perimeter |
|---|---|---|---|---|
| A | 5 in. | 6 in. | ■ | ■ |
| B | 4 in. | 13 in. | ■ | ■ |
| C | $6\frac{1}{2}$ in. | 8 in. | ■ | ■ |

**For Exercises 22 and 23, find the area and perimeter of each figure. Figures are not drawn to scale.**

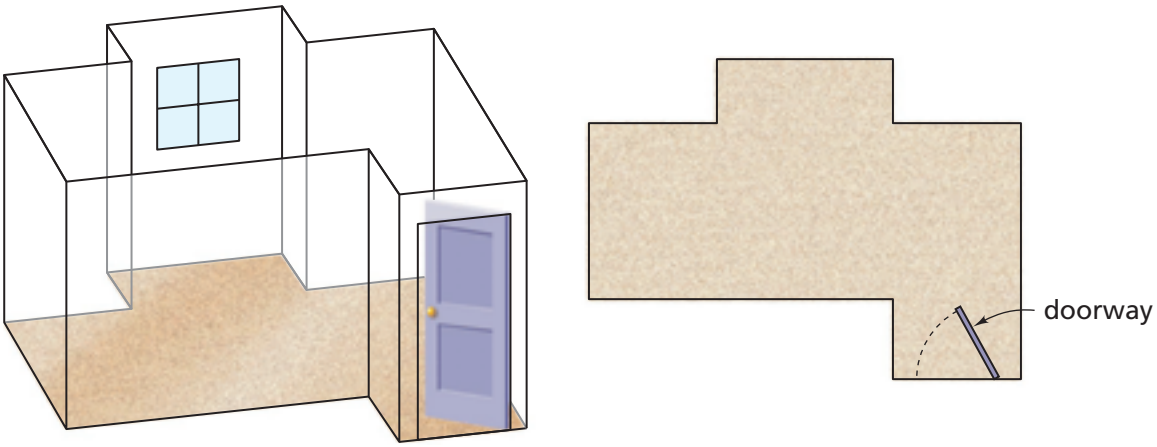
**22.**




**23.**



**24.** Carpet is usually sold by the square yard. Base molding, which is the strips of wood along the floor of the wall, is usually sold by the foot.




**a.** Describe a method you could use to compute the cost of carpet for the room sketched here.

**b.** Describe a method you could use to compute the cost of base molding around the base of the walls of this room.

**25.** Karl and Rita are building a playhouse for their daughter. The floor of the playhouse will be a rectangle that is 6 feet by $8\frac{1}{2}$ feet.

**a.** How much carpeting do Karl and Rita need to cover the floor?

**b.** How much molding do they need around the edges of the floor?

**c.** The walls will be 6 feet high. A pint of paint covers about 50 square feet. How much paint do they need to paint the inside walls? Explain.

**d.** Make your own plan for a playhouse. Figure out how much carpeting, wood, paint, and molding you would need to build the playhouse.

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**For:** Help with Exercise 25
**Web Code:** ame-5125

**26.** MARS sells a deluxe model of bumper-car rides for $95 per square foot. Most rides require about 100 square feet per bumper car. One ride design is a rectangle that is 40 feet by 120 feet.

**a.** How much does it cost to buy this design without cars?

**b.** What is the maximum number of cars this design can have?

**27.** Chuck and Ruth think that you can find the perimeter of a rectangle if you know its length and width. They each write a rule for finding the perimeter $P$ in terms of the length $\ell$ and the width $w$. Is either rule correct? Explain your reasoning.

Chuck's rule: $P = (2 \times \ell) + (2 \times w)$

Ruth's rule: $P = 2 \times (\ell + w)$

## Connections

**28.** **Multiple Choice** How many square feet are in one square yard?

**A.** 1 **B.** 3 **C.** 9 **D.** 27

**29.** Describe a flat surface in your home or classroom with an area of about one square foot. Describe another one with an area of about one square yard.

**30.** Which measure is greater? Or are the measures the same? Explain.

**a.** one square yard or one square foot

**b.** 5 feet or 60 inches

**c.** 12 meters or 120 centimeters

**d.** 12 yards or 120 feet

**e.** 50 centimeters or 500 millimeters

**f.** one square meter or one square yard

**31.** Sketch all the rectangles with whole-number dimensions for each area on grid paper.

**a.** 18 square units **b.** 25 square units **c.** 23 square units

**d.** Explain how the factors of a number are related to the rectangles you sketched for parts (a)–(c).

**32.** Find each product.

**a.** $4\frac{1}{4} \times 7\frac{2}{5}$ **b.** $12\frac{1}{2} \times 4$ **c.** $10\frac{5}{8} \times 2\frac{1}{4}$ **d.** $\frac{15}{6} \times \frac{7}{12}$

**33.** The product of two numbers is 20.

**a.** Suppose one number is $2\frac{1}{2}$. What is the other number?

**b.** Suppose one number is $1\frac{1}{4}$. What is the other number?

**c.** Suppose one number is $3\frac{1}{3}$. What is the other number?

**34.** Midge and Jon are making a pan of brownies. They use a 10 inch-by-10 inch baking pan. They want to cut the brownies into equal-sized pieces. For each possibility, give the dimensions of one piece. Sketch the cuts you would make to get the given number of brownies.

**a.** 25 pieces **b.** 20 pieces **c.** 30 pieces

**35.** **a.** What is the area of the bottom of the largest brownie from parts (a)–(c) of Exercise 34?

**b.** What is the area of the bottom of the smallest brownie from parts (a)–(c) of Exercise 34?

**36.** A football field is a rectangle 100 yards long and 50 yards wide (not counting the end zones).

**a.** What is the area of the football field in square yards? What is the perimeter in yards?

**b.** What is the area of the football field in square feet? What is the perimeter in feet?

**c.** John's classroom measures 20 feet by 25 feet. About how many classrooms will fit on one football field?

**37.** One soccer field is a rectangle 375 feet long and 230 feet wide.

**a.** What is the area of the soccer field in square feet? What is the perimeter in feet?

**b.** What is the area of the soccer field in square yards? What is the perimeter in yards?

**c.** Jamilla's classroom measures 15 feet by 25 feet. About how many classrooms will fit on this soccer field?

**38.** Copy and complete the table for rectangles with an area of 20 square feet.

**Rectangle Dimensions**

| Length (ft) | Width (ft) |
|---|---|
| 20 | 1 |
| 10 | 2 |
| 5 | 4 |
| $2\frac{1}{2}$ | 8 |
| ■ | ■ |
| ■ | ■ |

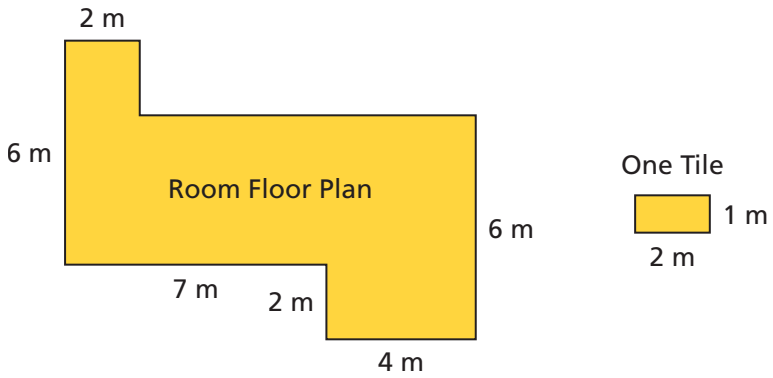
## Extensions

**39.** A group of students is finding the perimeters of rectangles whose lengths and widths are whole numbers. They notice that all the perimeters are even numbers. Is this always true? Explain why or why not.

**40.** Design a rectangle with an area of 18 square centimeters such that its length is twice its width.

**41.** Suppose you know the perimeter of a rectangle. Can you find its area? Explain why or why not.

**42.** How many rectangular tiles are needed to cover this floor?



# Mathematical Reflections 1

**In this investigation, you examined the areas and perimeters of figures made from square tiles. You found that some arrangements of tiles have large perimeters and other arrangements of the same tiles have smaller perimeters. You also found an efficient way to find the area and perimeter of a rectangle. These questions will help you to summarize what you have learned.**

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Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. Explain what area and perimeter of a figure refer to.
2. Is it possible for two shapes to have the same area but different perimeters? Explain your answer using words and drawings.
3. Describe how you can find the area of a rectangle if you know its length and width. Explain why this method works.
4. Describe how you can find the perimeter of a rectangle if you know its length and width. Explain why this method works.