

Changing Area, Changing Perimeter

Whether you make a floor plan for a bumper-car ride or a house, there are many options.

You should consider the cost of materials and the use of a space to find the best possible plan. In Investigation 1, you saw that floor plans with the same area could have different perimeters. Sometimes you want the largest, or *maximum*, possible area or perimeter. At other times, you want the smallest, or *minimum*, area or perimeter.

This investigation explores these two kinds of problems. You will find the maximum and minimum perimeter for a fixed area. You will also find the maximum and minimum area for a fixed perimeter. *Fixed* area or perimeter means that the measurement is given and does not change.

2.1 Building Storm Shelters

Sometimes, during a fierce winter storm, people are stranded in the snow, far from shelter. To prepare for this kind of emergency, parks often provide shelters at points along major hiking trails. Because the shelters are only for emergency use, they are designed to be simple buildings that are easy to maintain.



Problem 2.1 Constant Area, Changing Perimeter

The rangers in a national park want to build several storm shelters. The shelters must have 24 square meters of rectangular floor space.



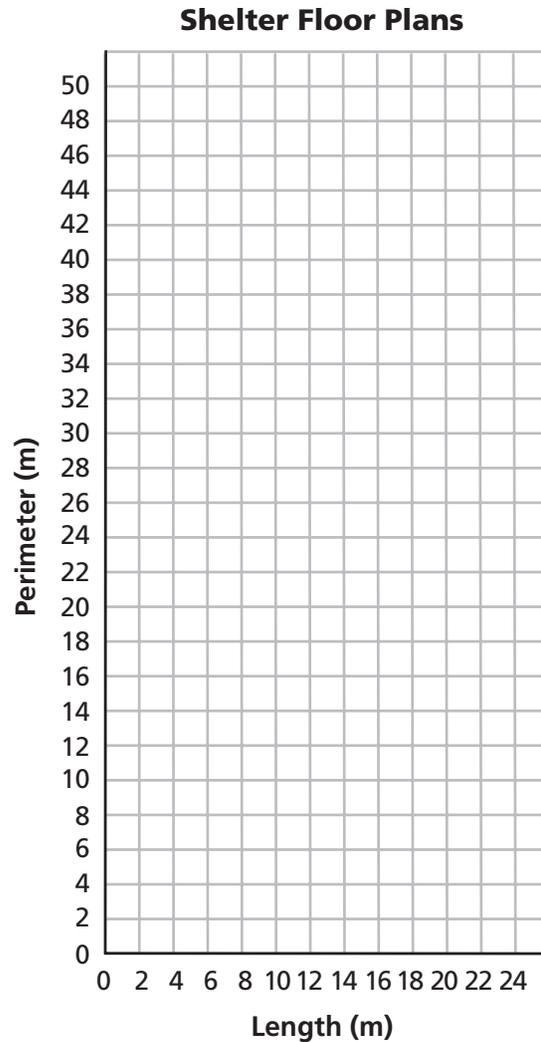
- A.** Experiment with different rectangles that have whole-number dimensions. Sketch each possible floor plan on grid paper. Record your data in a table such as the one started below. Look for patterns in the data.

Shelter Floor Plans

Length	Width	Perimeter	Area
1 m	24 m	50 m	24 sq. m

- B.** Suppose the walls are made of flat rectangular panels that are 1 meter wide and have the needed height.
1. What determines how many wall panels are needed, area or perimeter? Explain.
 2. Which design would require the most panels? Explain.
 3. Which design would require the fewest panels? Explain.

- C. 1. Use axes like the ones below to make a graph for various rectangles with an area of 24 square meters.



2. Describe the graph. How do the patterns that you observed in your table show up in the graph?
- D. 1. Suppose you consider a rectangular floor space of 36 square meters with whole-number side lengths. Which design has the least perimeter? Which has the greatest perimeter? Explain your reasoning.
2. In general, describe the rectangle with whole-number dimensions that has the greatest perimeter for a fixed area. Which rectangle has the least perimeter for a fixed area?

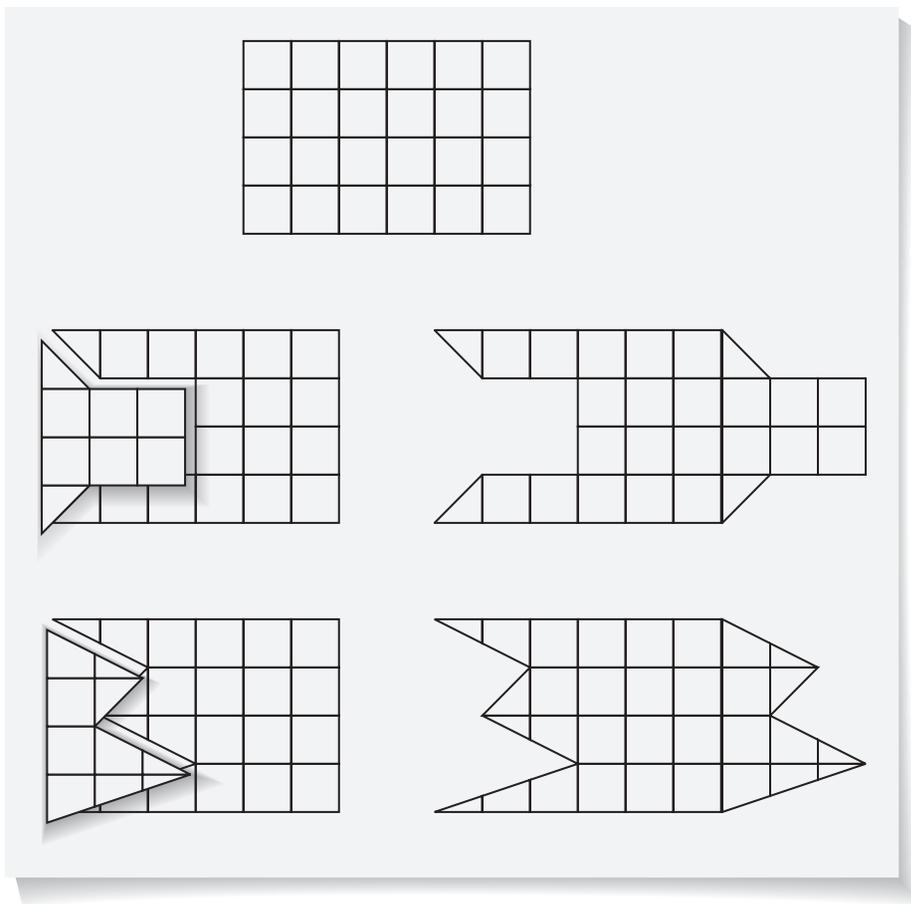
AC Homework starts on page 26.

2.2

Stretching the Perimeter

Getting Ready for Problem 2.2

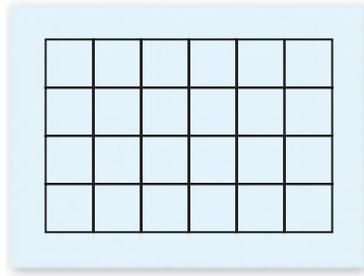
What happens to the perimeter of a rectangle when you cut a part from it and slide that part onto another edge? Here are some examples.



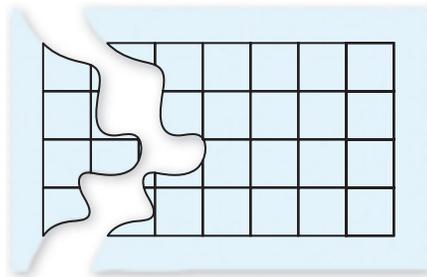
Think about whether you can use this technique to make nonrectangular shapes from a 4-by-6 rectangle to make a larger perimeter.

Problem 2.2 Perimeters and Irregular Shapes

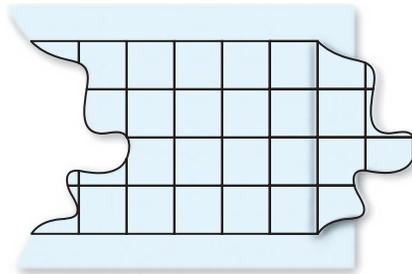
Draw a 4-by-6 rectangle on grid paper, and cut it out.



Starting at one corner, cut an interesting path to an adjacent corner.



Tape the piece you cut off to the opposite edge, matching the straight edges.



- Estimate the area and the perimeter of your new figure.
- Is the perimeter of the new figure greater than, the same as, or less than the perimeter of a 4-by-6 rectangle?
- Is the area of the new figure greater than, the same as, or less than the area of a 4-by-6 rectangle?
- Talecia asks, "Wait a minute! Can't you find the perimeter if you know the area of a figure?" How would you answer Talecia?
- Can you make a figure with an area of 24 square units that has a longer perimeter than the one you made? Explain your answer.

ACE Homework starts on page 26.

2.3

Fencing in Spaces

Americans have over 61 million dogs as pets. In many parts of the country, particularly in cities, there are laws against letting dogs run free. Many people build pens so their dogs can get outside for fresh air and exercise.

Problem 2.3 Constant Perimeter, Changing Area

Suppose you want to help a friend build a rectangular pen for her dog. You have 24 meters of fencing, in 1-meter lengths, to build the pen.

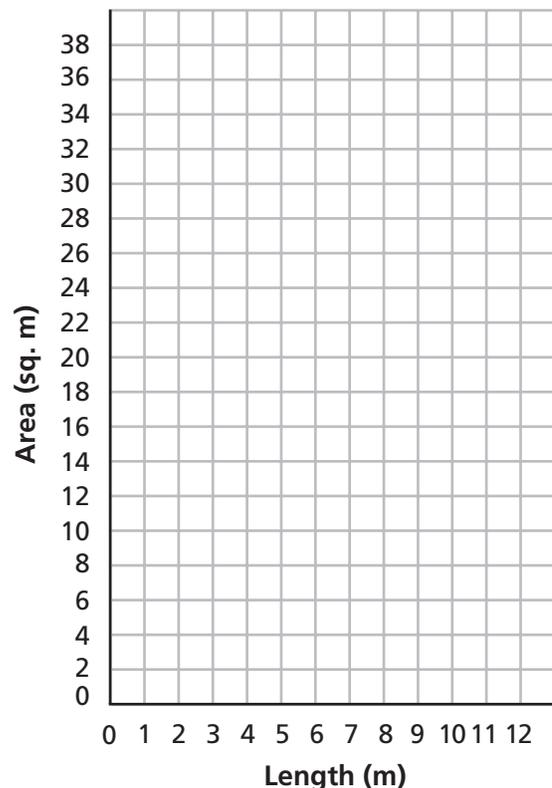
- A. 1.** Use tiles or grid paper to find all rectangles with whole-number dimensions that have a perimeter of 24 meters. Sketch each one on grid paper. Record your data about each possible plan in a table such as the one started below. Look for patterns in the data.

Dog Pen Floor Plans

Length	Width	Perimeter	Area
1 m	11 m	24 m	11 sq. m

- 2.** Which rectangle has the least area?
Which rectangle has the greatest area?
- B. 1.** Make a graph from your table, using axes similar to those at the right.
- 2.** Describe the graph. How do the patterns that you saw in your table show up in the graph?
- 3.** Compare this graph to the graph you made in Problem 2.1.
- C.** Suppose you have 36 meters of fencing. Which rectangle with whole-number dimensions has the least area? Which rectangle has the greatest area?
- D.** In general, describe the rectangle that has the least area for a fixed perimeter. Which rectangle has the greatest area for a fixed perimeter?

Dog Pen Floor Plans



ACE Homework starts on page 26.

2.4

Adding Tiles to Pentominos

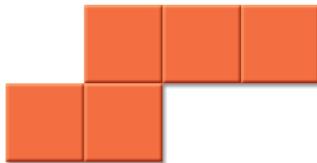
Shapes that are not rectangles can also be made from tiles. A *pentomino* (pen TAWM in oh) is a shape made from five identical square tiles connected along their edges. Turning or flipping a pentomino does not make a different pentomino, so these two figures are considered the same.



In this problem, you will add tiles to a pentomino and examine its area and perimeter.

Problem 2.4 Increasing Area and Perimeter

Make this pentomino with your tiles:



- Add tiles to the pentomino to make a new figure with a perimeter of 18 units. Draw the new figure on grid paper. Show where you added tiles to the pentomino.
- What is the fewest number of tiles you can add to the pentomino to make a new figure with a perimeter of 18 units? Draw the new figure, showing where you would add tiles to the pentomino.
- What is the greatest number of tiles you can add to the pentomino to make a new figure with a perimeter of 18 units? Draw the new figure, showing where you would add tiles to the pentomino.

ACE Homework starts on page 26.

Applications

1. Nu is designing a rectangular sandbox. The bottom is 16 square feet. Which dimensions require the least amount of material for the sides of the sandbox?
2. Alyssa is designing a garage with a rectangular floor area of 240 square feet.
 - a. List the length and width in feet of all the possible garages Alyssa could make. Use whole-number dimensions.
 - b. Which rectangles are reasonable for a garage floor? Explain.



**In Exercises 3–5, the area of a rectangle is given.
For each area, follow the steps below.**

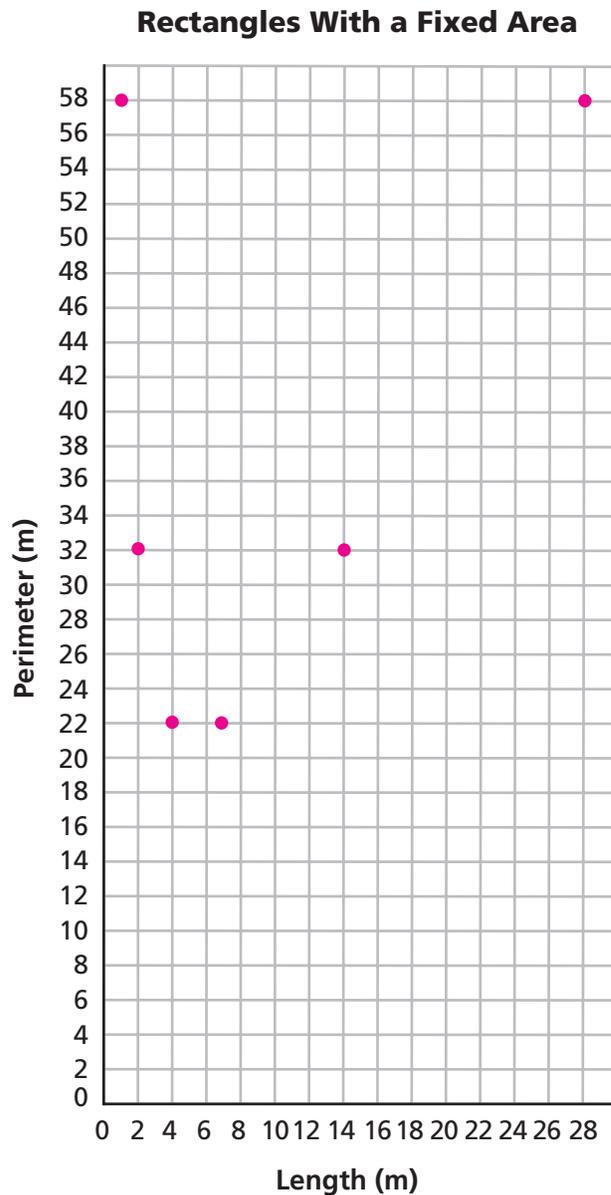
- a. Sketch all the rectangles with the given area and whole-number side lengths. Record the length, width, area, and perimeter in a table.
 - b. Sketch a graph of the length and perimeter.
 - c. Describe how you can use the table and graph to find the rectangle with the greatest perimeter and the rectangle with the least perimeter for Exercise 3.
3. 30 square meters
 4. 20 square meters
 5. 64 square meters

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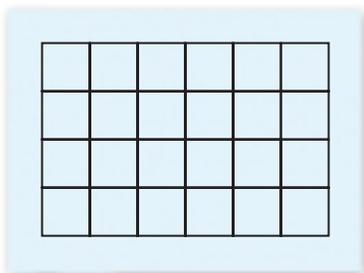
6. The graph shows the lengths and perimeters for rectangles with a fixed area and whole-number dimensions.



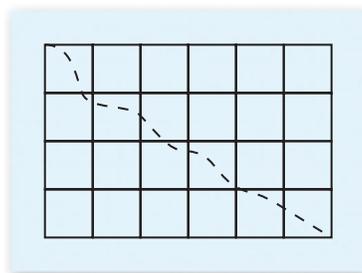
- What is the perimeter of a rectangle with a length of 2 meters?
What is its width?
- Describe the rectangle that has the greatest perimeter represented in the graph above.
- Describe the rectangle that has the least perimeter represented in the graph above.
- What is the fixed area? Explain how you found your answer.

7. Billie drew a 4-by-6 rectangle on grid paper. She started at an edge and cut a path to the opposite corner. Then she slid the piece onto the opposite edge, making the straight edges match.

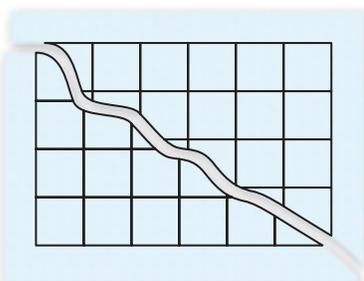
Step 1



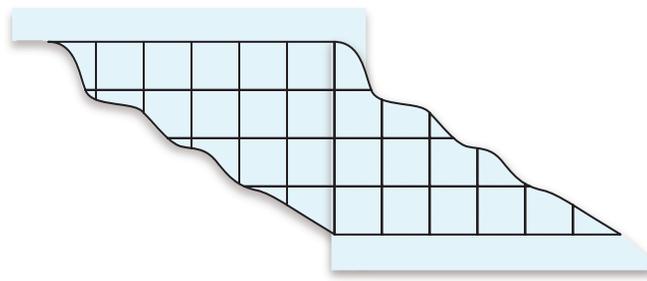
Step 2



Step 3



Step 4

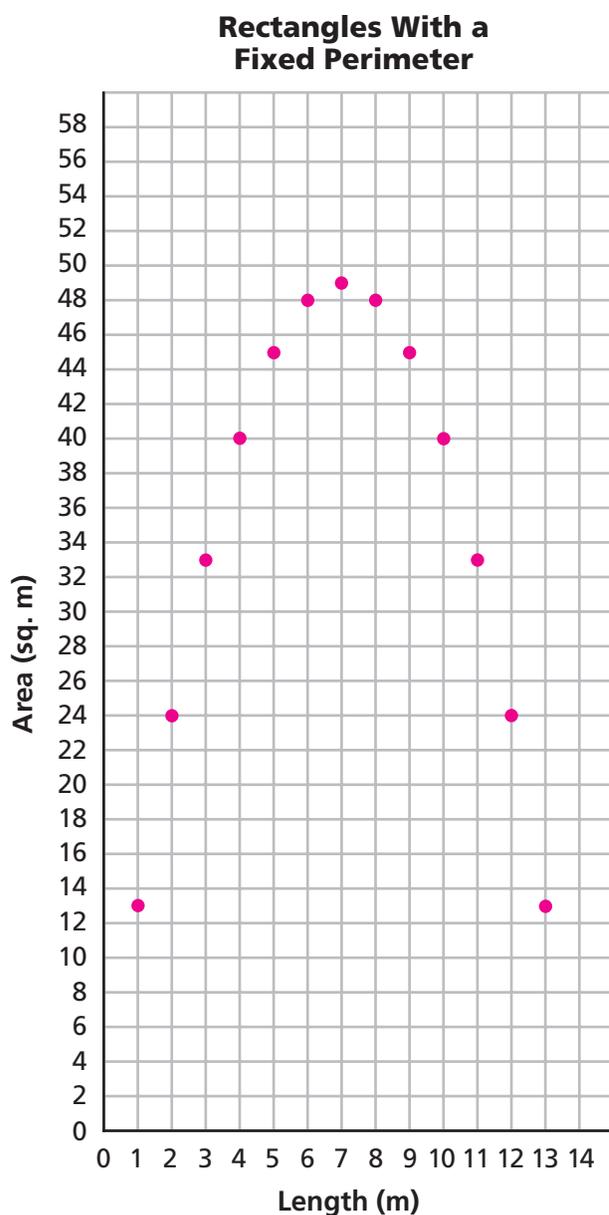


Are the area and perimeter of her new figure the same as, less than, or greater than the area and perimeter of the original figure? Explain how you found your answer.

8. Niran has 72 centimeters of molding to make a frame for a print. This is not enough molding to frame the entire print. How should he cut the molding to give the largest possible area for the print using the inside edge of the molding as the perimeter?



9. The graph below shows the whole-number lengths and areas for rectangles with a fixed perimeter.



- What is the area of a rectangle with a length of 2 meters? What is its width?
- Describe the rectangle that has the greatest area represented in the graph above.
- Describe the rectangle that has the least area represented in the graph above.
- What is the fixed perimeter? Explain.

In Exercises 10–12, the perimeter of a rectangle is given. For each perimeter, follow the steps below.

- a. Sketch all the rectangles with the given perimeter and whole-number side lengths. Record the length, width, area, and perimeter in a table.
- b. Sketch a graph of the length and area.
- c. Describe how you can use the table and graph to find the rectangle with whole-number dimensions that has the greatest area and the rectangle with the least area.

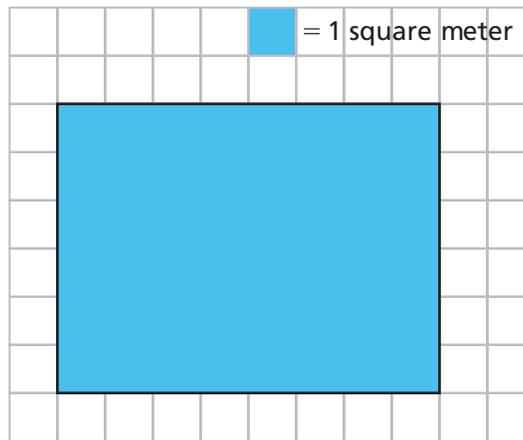
10. 8 meters

11. 20 meters

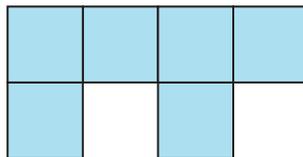
12. 15 meters

13. Diego says, “You can find the perimeter if you know the area of a rectangle.” Do you agree?

- 14. a.** Find the perimeter and area of the blue rectangle.
- b.** On grid paper, draw a rectangle with the same area as in part (a), but with a different perimeter. Label its dimensions and give its perimeter.
- c.** On grid paper, draw a rectangle with the same perimeter as the rectangle you just drew, but a different area. Label its dimensions and give its area.



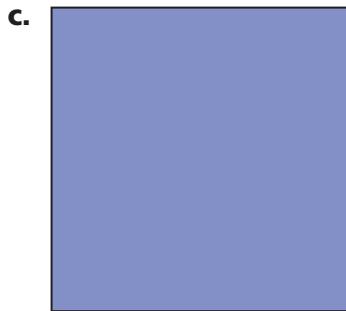
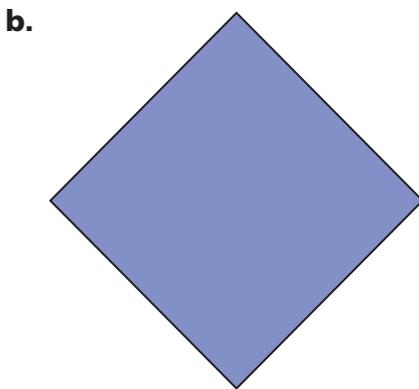
15. Multiple Choice Each tile in this figure is 1 square centimeter. Which result is impossible to get by adding one tile to this figure?



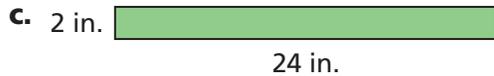
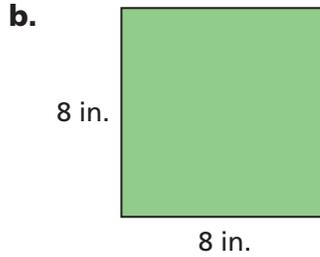
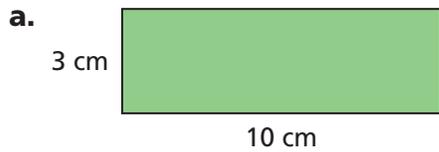
- A.** Increase the area by 1 square centimeter and the perimeter by 1 centimeter.
- B.** Increase the area by 1 square centimeter and the perimeter remains the same.
- C.** Increase the area by 1 square centimeter and decrease the perimeter by 2 centimeters.
- D.** Increase the area by 1 square centimeter and the perimeter by 2 centimeters.

Connections

- 16. a.** The floor area of a rectangular storm shelter is 65 square meters, and its length is $6\frac{1}{2}$ meters. What is the width of the storm shelter?
- b.** What is its perimeter?
- c.** A one-meter wall panel costs \$129.99. Use benchmarks to estimate the total cost of the wall panels for this four-sided shelter.
- 17. Multiple Choice** The area of a storm shelter is 24 square meters. The length is $5\frac{1}{3}$ meters. What is the width of the storm shelter in meters?
- F.** $4\frac{1}{2}$ **G.** $4\frac{1}{3}$ **H.** $4\frac{1}{4}$ **J.** $4\frac{1}{5}$
- 18.** These sketches show rectangles without measurements or grid background. Use a centimeter ruler to make any measurements you need to find the perimeter and area of each figure.



19. Find the area and perimeter of each rectangle.



20. Suppose fractional side lengths are allowed for the storm shelters in Problem 2.1. Is it possible to have a rectangle that has a smaller perimeter than the rectangles in your table? Explain.

21. **Multiple Choice** The perimeter of a dog pen is 24 meters. The length is $5\frac{1}{3}$ meters. What is the width of the dog pen in meters?

A. 6

B. $6\frac{1}{3}$

C. $6\frac{2}{3}$

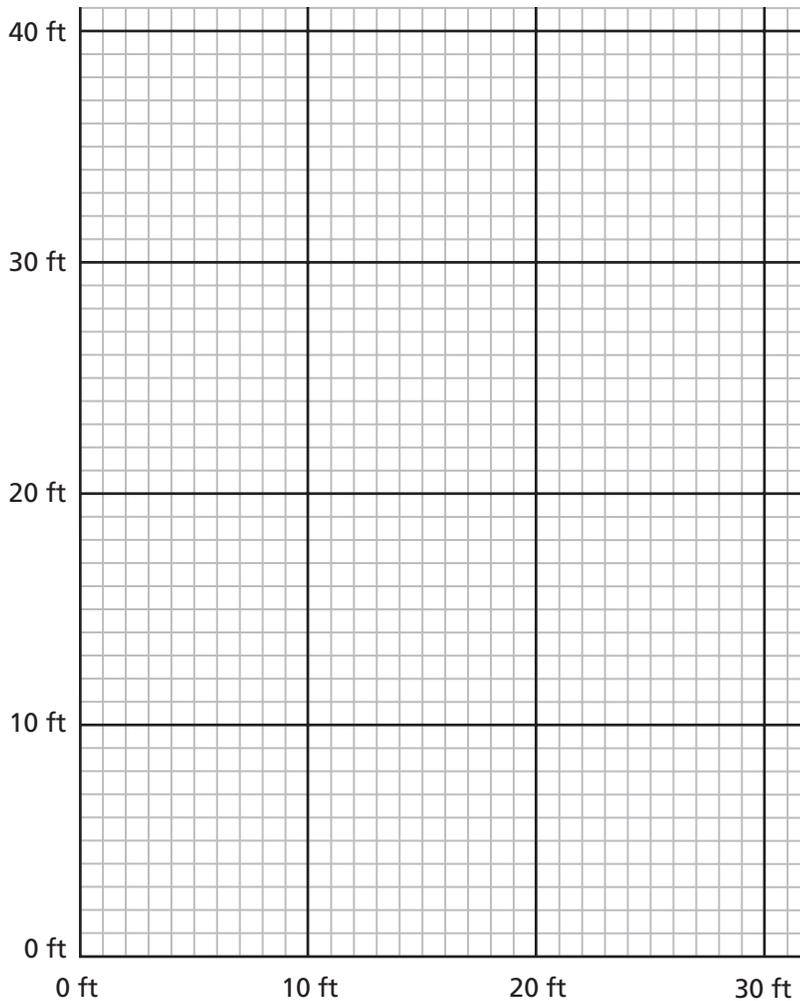
D. 7

22. Suppose fractional side lengths are allowed for the dog pens in Problem 2.3. Is it possible to have a rectangle that has a greater area than the rectangles in your table? Explain.

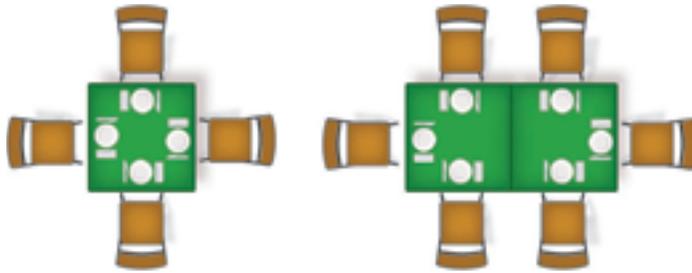


- 23.** The diagram below represents a field next to Sarah's house. Each small square shows a space that is one foot on each side.
- How many feet of fencing will Sarah need to enclose the field?
 - Each box of grass seed seeds an area of 125 square feet. How many boxes of seed will Sarah need to seed the field? Explain.
 - Sarah decides to include some flower and vegetable plots in the field, as well as a swing and a sandbox for her children. On grid paper, make a design for Sarah with these items. Give the area and the dimensions of each part of your design.
 - How many boxes of grass seed will she need to seed the new design?
 - What fraction of the area of the field can be covered with 1 box of grass seed?

Sarah's Field



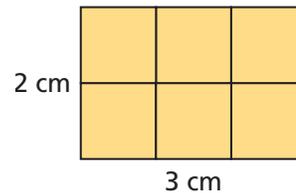
- 24.** Four people can be seated for dinner at a card table, one person on each side. With two card tables put together, six people can be seated.



How would you arrange 36 card tables to make a rectangular banquet table that seats the greatest number of people? Explain.

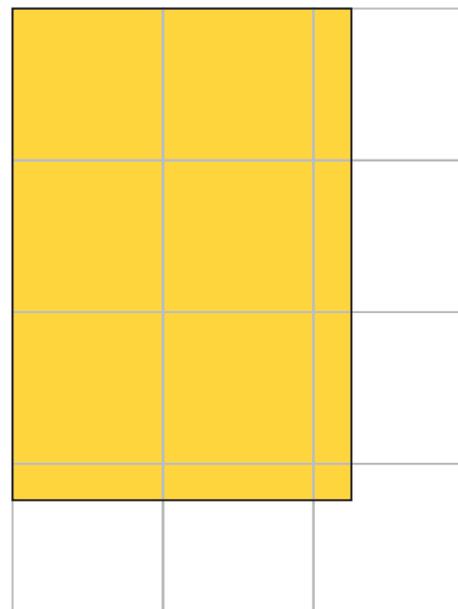
- 25.** For parts (a)–(c), find all the rectangles that can be made from the given number of square tiles.
- a.** 60 **b.** 61 **c.** 62
- d.** How can you use your work in parts (a)–(c) to list the factors of 60, 61, and 62?

- 26.** In the figure, each tile is 1 square centimeter. Remove one tile and sketch a figure that would represent a decrease of 1 square centimeter of area and an increase of 2 centimeters of perimeter.



Extensions

- 27. a.** Use a centimeter ruler. Find the perimeter and area of the shaded rectangle.
- b.** Draw another rectangle on grid paper that has the same perimeter as the one above but a different area. What is the area of the one you drew? Be sure to label the length and width.
- 28. a.** Find all the possible pentominoes. Sketch them on grid paper.
- b.** Why do you think you have found all the possible pentominoes?
- c.** Which pentomino has the least perimeter? Which pentomino has the greatest perimeter?



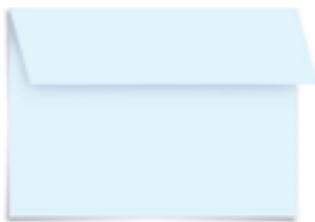
29. Suppose a square sheet of paper has a perimeter of 1 meter.



- a. What is the length of each side (in meters)?
- b. Suppose you fold the square sheet in half. What new shape would you have? What would the lengths of the shape's four sides be in meters? What would the perimeter be?

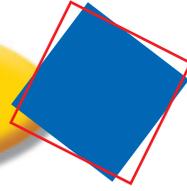


- c. Suppose you fold over the top $\frac{1}{4}$ of the square. What new shape do you have? What are the lengths of the shape's four sides in meters? What is the perimeter?



- d. Suppose you fold over only the top $\frac{1}{8}$ of the square. What new shape do you have? What are the lengths of the shape's four sides in meters? What is the perimeter?
- e. What do you predict for the perimeter of the shape if you fold over $\frac{1}{16}$ of the square?

Mathematical Reflections



In this investigation, you examined how shapes with the same perimeter can have different areas and how shapes with the same area can have different perimeters. These questions will help you to summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1.
 - a. Of all rectangles with whole-number dimensions that have a given area, how would you describe the one that has the least perimeter?
 - b. Of all rectangles with whole-number dimensions that have a given area, how would you describe the one that has the greatest perimeter?
2.
 - a. Of all rectangles with whole-number dimensions that have a given perimeter, how would you describe the one that has the least area?
 - b. Of all rectangles with whole-number dimensions that have a given perimeter, how would you describe the one that has the greatest area?