Investigation 1 Additional Practice

1. a. 36 fractions: \( \frac{1}{1} \), \( \frac{1}{2} + \frac{1}{2} \), \( \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \), \( \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \), \( \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} \), \( \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \).

   b. \( \frac{1}{12} + \frac{3}{12} + \frac{2}{12} + \frac{1}{12} \).

   c. \( \frac{1}{3} \) and \( \frac{1}{2} \).

2. a. \( \frac{3}{8} + \frac{2}{3} < 1 \) b. \( \frac{5}{10} + \frac{3}{4} > 1 \) c. \( \frac{3}{12} + \frac{3}{6} < 1 \)
   
   d. \( \frac{1}{2} + \frac{4}{8} = 1 \) e. \( \frac{4}{7} + \frac{7}{12} > 1 \) f. \( \frac{4}{3} + \frac{1}{100} > 1 \)

   g. \( \frac{1}{4} + \frac{2}{3} < 1 \) h. \( \frac{9}{20} + \frac{5}{11} < 1 \) i. \( \frac{9}{12} + \frac{2}{8} = 1 \)

3. a. \( \frac{1}{3} \); since \( \frac{1}{3} \) is less than \( \frac{1}{2} \), adding \( \frac{1}{2} \) will keep the sum less than 1. The other two options are less than \( \frac{1}{2} \); so those sums would not be as close to 1 as the sum \( \frac{1}{3} + \frac{1}{2} \).

   b. \( \frac{3}{8} + \frac{1}{3} \) is greater than 1 and \( \frac{3}{4} + \frac{1}{2} \) is greater than 1. Since \( \frac{1}{8} \) is less than \( \frac{1}{3} \), the sum \( \frac{3}{4} + \frac{1}{8} \) would be less than 1.

   c. \( \frac{1}{2} + \frac{3}{5} \) is greater than 1. Both \( \frac{1}{4} \) and \( \frac{1}{2} \) when added to \( \frac{3}{5} \) will be less than 1 but \( \frac{1}{2} + \frac{3}{5} \) will be closest to one without going over one.

   d. No solution. Since all the fractions are greater than \( \frac{1}{2} \), all of the sums are greater than 1.

   e. \( \frac{1}{4} \); all of the fractions can be added to \( \frac{1}{10} \) so that the sum is less than 1. Since \( \frac{1}{4} \) is the greatest of the options, it will give the largest sum that is less than 1.

   f. \( \frac{4}{9} + \frac{3}{8} \) is less than \( \frac{1}{2} \) and \( \frac{4}{9} \) is less than \( \frac{1}{2} \) so \( \frac{3}{8} + \frac{4}{9} \) is less than 1. The other two choices would give a sum greater than 1.

4. a. \( \frac{1}{2} \) and \( \frac{3}{4} \) and \( \frac{7}{8} \)
   
   b. \( \frac{1}{3} \) and \( \frac{9}{10} \)

   c. None; all sums are too great.

   d. \( \frac{1}{3} \) and \( \frac{3}{5} \)

   e. \( \frac{1}{10} \) and \( \frac{5}{4} \)

   f. \( \frac{1}{2} \) and \( \frac{3}{5} \) and \( \frac{7}{10} \) and \( \frac{3}{5} \) and \( \frac{7}{10} \)

5. a. $25

   b. It would be better to overestimate to make sure they have enough money.

Skill: Estimating With Fractions

1. \( \frac{5}{8} \), \( \frac{1}{8} \), \( \frac{9}{10} \), \( \frac{11}{12} \), \( \frac{1}{2} \), \( \frac{5}{12} \)

2. \( \frac{9}{10} \), \( \frac{1}{2} \), \( \frac{1}{8} \), \( \frac{1}{3} \), \( \frac{1}{10} \), \( \frac{1}{2} \), \( \frac{1}{5} \), \( \frac{1}{4} \), \( \frac{1}{2} \)

3. \( \frac{1}{2} \), \( \frac{1}{3} \), \( \frac{1}{4} \), \( \frac{1}{5} \), \( \frac{1}{6} \), \( \frac{1}{7} \), \( \frac{1}{8} \), \( \frac{1}{9} \), \( \frac{1}{10} \)

15. Sample answer: \( \frac{3}{8} \), \( \frac{2}{5} \), \( \frac{1}{3} \)

16. Sample answer: \( \frac{1}{8} \), \( \frac{2}{10} \), \( \frac{1}{12} \)

Skill: Estimating With Mixed Numbers

1. \( \frac{9}{10} \), \( \frac{1}{2} \), \( \frac{13}{15} \), \( \frac{7}{12} \), \( \frac{25}{30} \), \( \frac{5}{6} \), \( \frac{8}{10} \), \( \frac{7}{12} \)

2. \( \frac{1}{2} \), \( \frac{3}{4} \), \( \frac{1}{4} \), \( \frac{5}{6} \), \( \frac{7}{8} \), \( \frac{1}{3} \), \( \frac{2}{4} \), \( \frac{1}{5} \), \( \frac{3}{10} \), \( \frac{1}{2} \), \( \frac{1}{6} \), \( \frac{1}{8} \)

11. \( \frac{9}{10} \), \( \frac{1}{2} \), \( \frac{1}{3} \), \( \frac{1}{4} \), \( \frac{1}{5} \), \( \frac{1}{6} \), \( \frac{1}{7} \), \( \frac{1}{8} \), \( \frac{1}{9} \), \( \frac{1}{10} \)

16. \( \frac{6}{12} \), \( \frac{17}{24} \), \( \frac{3}{12} \), \( \frac{19}{20} \), \( \frac{6}{10} \), \( \frac{20}{22} \)

21. \$34 or $34.8

Investigation 2 Additional Practice

1. a. \( \frac{1}{8} \) cups

   b. i. \( \frac{1}{2} \) ii. \( \frac{2}{3} \) iii. \( \frac{1}{8} \) c. \( \frac{1}{8} \)

2. a. \( \frac{1}{2} \)

   b. \( \frac{3}{8} \)

   c. Yes, there are enough seats, and \( \frac{1}{8} \) are left for open seating.

3. a. \( \frac{3}{4} + \frac{1}{5} - \frac{19}{20} \)

   b. \( \frac{7}{8} - \frac{1}{2} = \frac{3}{8} \)

   c. \( \frac{1}{6} + \frac{5}{12} = \frac{7}{12} \)

   d. \( \frac{11}{20} - \frac{1}{5} = \frac{7}{20} \)
e. $\frac{7}{5} - \frac{2}{3} = \frac{1}{9}$  

f. $\frac{3}{2} + \frac{3}{4} = \frac{9}{4}$  

g. $\frac{4}{3} + \frac{1}{6} = \frac{29}{30}$  

h. $\frac{1}{2} - \frac{3}{10} = \frac{1}{5}$  

i. $\frac{7}{3} + \frac{4}{21} = \frac{13}{21}$  

4. a. $1\frac{1}{2}$  

b. i. $\frac{3}{4}$ ii. $1\frac{1}{4}$ iii. $\frac{1}{8}$ iv. $\frac{5}{8}$  

c. $\frac{1}{3}$; it takes 3 of the shaded grey regions to fill the entire rectangle.  

5. a. $\frac{5}{6}$ b. $\frac{7}{12}$ c. $\frac{9}{12}$ d. $\frac{11}{30}$ e. $\frac{13}{42}$ f. $\frac{15}{56}$  

For the sum of two unit fractions whose denominators are consecutive integers, the numerator of the sum is the sum of the two denominators. The denominator of the sum is the product of the two denominators.

Skill: Adding and Subtracting Fractions  
1. $\frac{3}{4}$  

2. $\frac{3}{10}$  

3. $\frac{1}{4}$  

4. $\frac{3}{4}$  

5. $\frac{7}{8}$  

6. $\frac{9}{10}$  

7. $\frac{3}{10}$  

8. $\frac{3}{8}$  

9. $\frac{1}{10}$  

10. $\frac{5}{16}$  

11. $\frac{1}{2}$  

12. $\frac{3}{10}$  

13. $\frac{1}{2}$  

14. $\frac{1}{2}$  

15. $\frac{1}{3}$  

16. $\frac{5}{9}$  

Skill: Adding and Subtracting Mixed Numbers  
1. $9\frac{7}{10}$  

2. $6\frac{3}{8}$  

3. $8\frac{11}{12}$  

4. $9\frac{1}{4}$  

5. $4\frac{1}{4}$  

6. $19\frac{7}{10}$  

7. $13\frac{1}{4}$  

8. $15\frac{7}{12}$  

9. $7\frac{5}{12}$  

10. $6\frac{13}{16}$  

11. $5\frac{23}{24}$  

12. $5\frac{3}{5}$  

13. $2\frac{13}{16}$  

14. $4\frac{23}{30}$  

15. $4\frac{1}{2}$  

16. $1\frac{5}{8}$  

17. $2\frac{1}{16}$  

18. $5\frac{5}{6}$  

19. $74\frac{15}{16}$ pounds  

20. about $204$

Investigation 3 Additional Practice  
1. a. $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$  

b. $\frac{3}{5} \times \frac{10}{9} = \frac{2}{3}$  

c. $\frac{3}{4} \times \frac{2}{5} = \frac{3}{5}$  

d. $\frac{3}{2} \times \frac{5}{6} = \frac{5}{4}$  

e. $\frac{2}{3} \times \frac{1}{3} = \frac{2}{21}$  

f. $\frac{3}{8} \times \frac{12}{15} = \frac{3}{10}$  

g. $\frac{9}{10} \times \frac{1}{6} = \frac{3}{20}$  

h. $\frac{1}{2} \times \frac{6}{7} = \frac{3}{7}$  

i. $360 \times \frac{7}{9} = 280$  

2. a. 110  

b. 176  

c. $\frac{7}{20}$  

d. 154  

3. a. $A = \frac{1}{3}$, $B = \frac{1}{6}$, $C = \frac{1}{24}$, $D = \frac{1}{8}$, $E = \frac{1}{12}$, $F = \frac{1}{4}$  

d. $\frac{80}{c}$  

b. $\frac{30}{c}$  

d. $\frac{60}{d}$  

e. i. $\frac{70}{e}$ ii. $\frac{17}{24}$  

4. a. $\frac{3}{4}$ cup  

b. $\frac{1}{8}$ cup  

c. No; Jane is increasing and decreasing different numbers by half ($\frac{1}{4}$ and $\frac{1}{3}$), so the new sum will be different.  

5. a. $\frac{3}{20}$  

b. Possible Answers: Since Paul’s sister used $\frac{1}{4}$ of the roll, there is $\frac{3}{4}$ of $\frac{3}{5}$ of a roll left, so $\frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$. Also, since $\frac{3}{20}$ of a whole roll was used and there was $\frac{3}{5}$ of a whole roll initially, $\frac{3}{5} - \frac{3}{20} = \frac{9}{20}.$

6. a. Equal; both products are $\frac{2}{15}$.  

b. Equal; both products are $\frac{42}{40}$ or $1\frac{1}{20}$.  

c. Equal; both products are $\frac{150}{90}$ or $\frac{1}{6}$.  

d. Equal; both products are $\frac{15}{56}$.  

The numbers in the numerators and the denominators are the same in each problem but the placement of the numbers is different. Since you are multiplying the same numbers, the products are equal.  

7. about 1,338 pounds
Skill: Multiplying Fractions
1. \[
\begin{array}{ccc}
\hline
& \frac{1}{2} & \frac{1}{4} \\
\hline
\frac{1}{3} & \times & \frac{1}{3} \\
\hline
\end{array}
\]
2. \[
\begin{array}{ccc}
\hline
& \frac{1}{2} & \frac{1}{3} \\
\hline
\frac{1}{4} & \times & \frac{1}{4} \\
\hline
\end{array}
\]
3. \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}
4. \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}
5. \frac{4}{5} \times \frac{4}{5} = \frac{16}{25}
6. \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}
7. \frac{5}{16} \times \frac{3}{10} = \frac{15}{160} = \frac{3}{32}
8. \frac{9}{20} \times \frac{1}{6} = \frac{9}{120} = \frac{3}{40}
9. \frac{3}{32} \times \frac{3}{32} = \frac{9}{1024}
10. \frac{14}{55} \times \frac{1}{5} = \frac{14}{275}
11. \frac{1}{6} \times \frac{1}{5} = \frac{1}{30}
12. \frac{1}{5} \times \frac{1}{5} = \frac{1}{25}
13. \frac{1}{5} \times \frac{1}{5} = \frac{1}{25}
14. \frac{1}{5} \times \frac{1}{5} = \frac{1}{25}
15. \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}
16. \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}
17. \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}
18. \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}
19. \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}

Skill: Multiplying Mixed Numbers
1. \[\frac{23}{24} \times \frac{15}{32} = \frac{345}{768} = \frac{5}{4}
2. \[\frac{15}{20} \times \frac{7}{20} = \frac{105}{400} = \frac{21}{80}
3. \[\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}
4. \[\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}
5. \[\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}
6. \[\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}
7. \[\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}
8. \[\frac{1}{8} \times \frac{1}{8} = \frac{1}{64}
9. \[\frac{1}{9} \times \frac{1}{9} = \frac{1}{81}
10. \[\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}
11. \[\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}

Investigation 4 Additional Practice
1. a. \[20 \div 2 = 10\] b. \[20 \div 1 = 20\] c. \[20 \div \frac{1}{2} = 40\] d. \[20 \div \frac{1}{4} = 80\] e. \[20 \div \frac{1}{8} = 160\]
   Each time the number you are dividing by is cut in half, the quotient or answer doubles. This happens because each time you cut the number you are dividing by in half, the amount you are dividing by is smaller, so twice as many will divide into 20.
2. milkshake: \[3 \div \frac{1}{8} = 24\] shakes
double milkshake: \[3 \div \frac{1}{4} = 12\] double shakes
   triple milkshake: \[3 \div \frac{3}{8} = 8\] triple shakes
3. C; the 4 students who share \(\frac{2}{3}\) of a pizza \(\left(\frac{2}{3} \div 4\right)\) will each get \(\frac{1}{6}\) of the pizza. If you divide the amount of leftover pizza by the number of students in the other groups, you get: Group A: \(\frac{3}{4} \div 6 = \frac{1}{8}\) of the pizza for each student. Group B: \(\frac{1}{3} \div 3 = \frac{1}{9}\) of the pizza for each student.
4. a. \[12 \div \frac{1}{2} = 24\] b. \[12 \div 1\frac{1}{3} = 36\] c. \[3 \div \frac{2}{3} = 4\frac{1}{2}\] d. \[\frac{7}{8} \div 4 = \frac{7}{32}\] e. \[\frac{2}{3} \div 6 = \frac{1}{9}\] f. \[\frac{5}{6} \div \frac{1}{3} = 2\frac{1}{2}\] g. \[\frac{1}{4} \div \frac{1}{2} = \frac{1}{2}\] h. \[\frac{8}{5} \div \frac{3}{10} = 5\frac{1}{3}\] i. \[\frac{1}{2} \div \frac{3}{4} = 2\]
5. When you divide fractions with common denominators, you only have to divide the numerators. The denominator is the size of the fraction you are working with. The numerator is the number of those size parts. For example, with \(\frac{6}{8}\) divided by \(\frac{3}{8}\), eighths are the size of the fractional parts that you have. You are trying to find out how many times three-eighths will go into six-eighths. This is the same as asking how many times 3 goes into 6.
6. a. Sam: 5 complete signs. Possible explanation: \(4 \div \frac{3}{4} = 5\frac{1}{3}\). That is, 5 complete signs and \(\frac{1}{3}\) of a sixth sign.
   Trish: 6 complete signs. Possible explanation: \(4 \div \frac{2}{3} = 6\) complete signs.
   Trish has no time left over.
   Shanti: 6 complete signs. Possible explanation: \(4 \div \frac{5}{6} = 6\frac{2}{3}\). That is, 6 complete signs and \(\frac{2}{3}\) of a seventh sign.
b. Shanti has the most time remaining.
   Possible explanation: Shanti has $\frac{2}{3}$ of $\frac{3}{5}$
of an hour left over. This is $\frac{2}{3} \times \frac{3}{5} =$
$\frac{2}{5}$ hour or 24 minutes left over. Sam has
$\frac{1}{3}$ of $\frac{3}{4}$ hour left over or $\frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$ of an
hour or 15 minutes left over. Trish has
no time left over.

7. 20 bows

Skill: Dividing Fractions

1.

2. $\frac{1}{10}$  3. 12  4. 8  5. $\frac{5}{9}$  6. 12
7. $\frac{2}{5}$  8. $\frac{7}{24}$  9. $\frac{1}{6}$  10. $\frac{1}{18}$  11. 3
12. $3\frac{1}{2}$  13. 10  14. 8 servings

Skill: Dividing Mixed Numbers

1. $2\frac{1}{4}$  2. 1  3. $\frac{9}{16}$  4. 2  5. $1\frac{1}{9}$
6. $2\frac{1}{3}$  7. $3\frac{2}{3}$  8. $2\frac{7}{16}$  9. $1\frac{4}{5}$  10. $1\frac{11}{19}$
11. $5\frac{3}{7}$  12. $4\frac{3}{8}$  13. $7\frac{1}{2}$ servings
14. 40 times  15. 24 min