Investigation 3

ACE Assignment Choices

Differentiated Instruction
Solutions for All Learners

Problem 3.1

Core 1–4, 28, 29
Other *Connections* 30

Problem 3.2

Core 5–14
Other *Connections* 31; unassigned choices from previous problems

Problem 3.3

Core 15–19, 22–24
Other *Applications* 20, 21; *Connections* 32–33; *Extensions* 34, 41–43; unassigned choices from previous problems

Problem 3.4

Core 27
Other *Applications* 25, 26; *Extensions* 35–40; unassigned choices from previous problems

Adapted For suggestions about adapting Exercises 1–4 and other ACE exercises, see the CMP *Special Needs Handbook*
Connecting to Prior Units 28, 32: *Bits and Pieces I;* 29: *Bits and Pieces II;* 33: *Covering and Surrounding*

Applications

1.–4. These are possible answers. Students may use other approaches. Check to be sure their explanations justify their choice of operation.

1. Subtraction; if you subtract the lower weight from the higher, you will find the difference between the two.

2. Division; you are finding how many $1\frac{1}{3}$ yards are in 6.5 yards.

3. Multiplication; you are adding the same value, \$2.95, three times. This is equivalent to multiplying by 3.

4. Division; you are finding how many 0.26 meter lengths will fit into 42 meters or the total length of walkway that is being bricked.

5. Possible diagram and explanation: (Figure 1) $2.6 \div 0.4$ can be thought of as how many times does 0.4 go into 2.6. The diagram shows that there are 6 whole 0.4's in 2.6. There is a remainder of 0.2, which is half of the dividend 0.4. The solution means that there are $6\frac{1}{2}$ or 6.5 sets of 0.4 in 2.6.

6. **a.** Greater than 1; possible explanation: The divisor is less than the dividend, meaning it will go in more than once.

b. Greater than 1; possible explanation: The divisor is less than the dividend, meaning it will go in more than once.

c. Less than 1; possible explanation: The divisor is larger than the dividend. If you think of the problem as approximately $5 \div 11$, it can mean how many 11's fit into 5. Since 11 is greater than 5, it will not fit into 5 an entire time. About half of 11 will fit into 5.

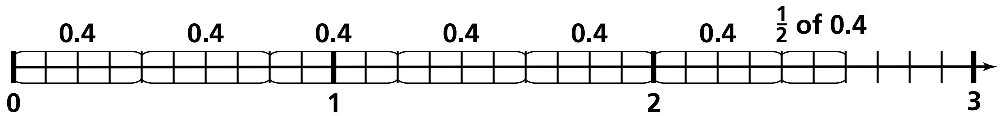
d. Less than 1; possible explanation: The divisor is larger than the dividend.

7. $\frac{45}{10} \div \frac{9}{10} = 45 \div 9 = 5$

8. $\frac{6}{10} \div \frac{12}{100} = \frac{60}{100} \div \frac{12}{100} = 60 \div 12 = 5$

9. $\frac{12}{10} \div \frac{5}{10} = 12 \div 5 = 2\frac{2}{5} = 2.4$

Figure 1



10. $\frac{18}{100} \div \frac{3}{100} = 18 \div 3 = 6$

11. $\frac{225}{10} \div \frac{15}{10} = 225 \div 15 = 15$

12. $\frac{342}{100} \div \frac{19}{100} = 342 \div 19 = 18$

13. $\frac{224}{10} \div \frac{5}{10} = 224 \div 5 = 44\frac{4}{5}$ or 44.8. The quotient means that five-tenths, or one-half, goes into 22.4 forty-four whole times and four-fifths of another whole more. If you had 22.4 feet of ribbon to make into bows that were 0.5 or $\frac{1}{2}$ ft in length, you would be able to make 44 complete bows and there would be enough ribbon left to make $\frac{4}{5}$, or 0.8, of another bow.

14. Possible answer: I can write $40.1 \div 0.5$ as $\frac{401}{10} \div \frac{5}{10}$. The answer to this fraction division problem is $401 \div 5$. So, the answer to the first problem is the same as the answer to the second.

15. 2.145

16. 361.04

17. 0.25

18. 0.199

19. **a.** 3 **b.** 0.3 **c.** 3
d. 30 **e.** 0.03 **f.** 30

20. **a.** 3.875 **b.** 3.875 **c.** 3.875
d. 3875 **e.** 0.3875 **f.** 3.875

21. **a.** 0.037
b. It has the same digits, but they are in different decimal places. The seven, for example, is in the thousandths place.
c. 0.0037. Again, it has the same digits but in different decimal places.
d. In general, when dividing a number by 10, the digits stay the same, but the value of each digit shifts to the value of the next decimal place to the right.

22. **a.** Possible answer: $4.8 \div 1.2$; and $0.048 \div 0.012$. Each problem can be renamed as $48 \div 12$. For example, $0.048 \div 0.012$ is 48 thousandths divided by 12 thousandths. The size of the unit is thousandths, but the division problem is asking how many times does 12 thousandths go into 48 thousandths, or $48 \div 12$.
b. Possible answer: $480 \div 12$ and $48 \div 1.2$; each problem can be renamed as 480 thousandths $\div$ 12 thousandths.

23. $N = 0.7$; $0.42 \div 0.7 = 0.6$, $0.42 \div 0.6 = 0.7$; $0.6 \times 0.7 = 0.42$; and $0.7 \times 0.6 = 0.42$

24. $N = 3.2$; $3.2 \div 0.5 = 6.4$; $3.2 \div 6.4 = 3.2$; $6.4 \times 0.5 = 3.2$; $0.5 \times 6.4 = 3.2$

25. **a.** 0.3333. . . **b.** 0.3333. . . **c.** 0.3333. . .
d. Each of the three fractions, $\frac{2}{6}$, $\frac{13}{39}$, and $\frac{5}{15}$, are equivalent to the fraction $\frac{1}{3}$, whose decimal equivalent is 0.3333. . . . Any fraction that is equivalent to $\frac{1}{3}$ will have the decimal equivalent 0.3333. . . .

26. **a.** 1.2222. . . **b.** 1.2222. . .
c. $\frac{11}{9}$ and $1\frac{6}{27}$ are equivalent to each other. They can both be expressed as $1\frac{2}{9}$ or the decimal equivalent 1.2222. . . .

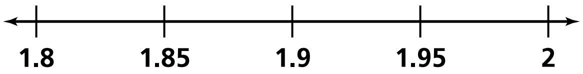
27. **a.**

Fraction	Decimal
$\frac{1}{9}$	0.111. . .
$\frac{2}{9}$	0.2222. . .
$\frac{3}{9}$	0.3333. . .
$\frac{4}{9}$	0.4444. . .
$\frac{5}{9}$	0.5555. . .
$\frac{6}{9}$	0.6666. . .
$\frac{7}{9}$	0.7777. . .
$\frac{8}{9}$	0.8888. . .

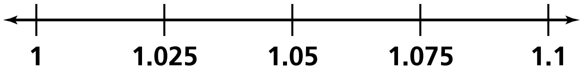
b. The numerator of the fraction repeats in the decimal, starting with the tenths place.
c. i. 1 (although the pattern suggests 0.999. . . in fact, the two answers are equal, $1 = 0.999\ldots$)
ii. 1.111. . . **iii.** 1.666. . .
d. i. $1\frac{2}{9}$ **ii.** $2\frac{7}{9}$

Connections

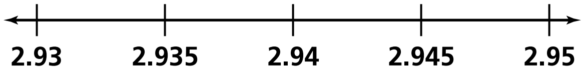
28. a.



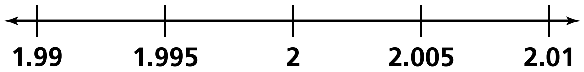
b.



c.



d.



e. Possible answer: I found the value of the middle mark, then found the middle of each of the lesser intervals.

29. D

30. a. Between 1968 and 1972, the score changed by 423.94 points.

b. Between 1976 and 1980, the score changed by 285.50 points.

c. Between 1996 and 2000, the score changed by 7.26 points.

d. 742.605 points

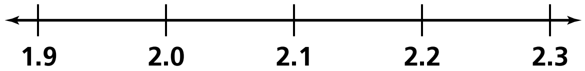
31. a. 3.7

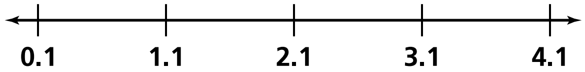
b. The digits are the same, but they are in different decimal places. The seven, for example, is now in the tenths place instead of the hundredths place.

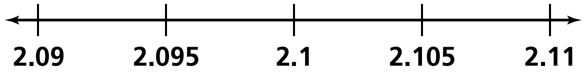
c. 37. Again, the digits are the same, but in different places.

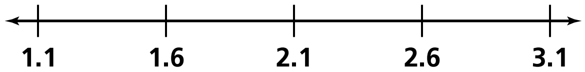
d. In general, when we multiply a decimal number by 10, the digits will stay the same but will shift to the next decimal place to the left.

32. a. Possible answers:









b. The mean is always 2.1.

c. No. In order for the sum of the 5 positive numbers to be 10, their mean would have to be 2. This is not possible.

33. a. 1.6 in.; Since $A = \pi r^2$ we have so find r when $A = 2.0096$. Dividing 2.0096 by π, we get 0.64 for r^2 so $r = 0.8$. Thus the diameter is 2(0.8), or 1.6 in.

b. 5.024 in.; Since $C = \pi d$ we have that $C = \pi(1.6)$, or 5.024 in.

Extensions

34. About 23.6 miles per gallon; since in 429.5 miles she used 18.2 gallons of gas, dividing 429.4 by 18.2 we get about 23.6 miles per gallon.

35. $\frac{1}{99} = 0.010101\ldots$

$\frac{2}{99} = 0.020202\ldots$

$\frac{45}{99} = 0.454545\ldots$

In general, there is a repeating pattern of two digits. The numerator is the repeating number if it is greater than 10. If the numerator is less than 10, the repeating digits are 0 and the numerator.

36. $\frac{1}{999} = 0.001001001\ldots$

$\frac{2}{999} = 0.002002002\ldots$

$\frac{45}{999} = 0.045045045\ldots$

$\frac{123}{999} = 0.123123123\ldots$

In general, there is a repeating pattern of three digits. The numerator is the repeating number if it is greater than 100. If the numerator is less than 100, the repeating digits are the numerator together with either one or two zeroes.

37. $\frac{5}{99}$ **38.** $\frac{45}{99}$ **39.** $\frac{45}{999}$ **40.** $10\frac{12}{99}$

41. 0.6 cm; the area of the original rectangle is $3.6 \times 1.2 = 4.32$ cm^2 so the area of this figure is also 4.32 cm^2. Since there are 12 squares, the area of each square must be 4.32 cm^2 $\div$ 12 or 0.36 cm^2. Thus each square must be 0.6 cm on a side in order for the area to be 0.36 cm^2 since $0.6 \times 0.6 = 0.36$. Hence the length of n is 0.6 cm.

42. 1.5 cm; since the area of the parallelogram is $A = bh$ and $A = 4.32$ cm^2 and $b = 2.88$ cm, the height is 4.32 cm^2 $\div$ 2.88 = 1.5 cm.

43. 2.25 cm; since the area of the triangle is $A = \frac{1}{2}bh$ with $A = 4.32$ cm^2 and $h = 3.84$ cm we get that $4.32 = \frac{1}{2}b(3.84)$. Using fractions we see that the equation is $\frac{432}{100} = \frac{50}{100} \times b \times \frac{384}{100}$ or $\frac{43{,}200}{10{,}000} = \frac{50}{100} \times b \times \frac{384}{100}$, so we need to find a number b such that $50 \times b \times 384$ is 43,200. If you divide 43,200 by 50 then this by 384 you get that b is 2.25 cm.

Possible Answers to Mathematical Reflections

1. Write the two decimal numbers as fractions. If the numbers are larger than 1, write improper fractions. Now, write these fractions with common denominators and whole-number numerators. The quotient of the numerators will be the same as the original quotient. For $0.4 \div 0.02$, we can write $\frac{4}{10} \div \frac{2}{100}$ and then $\frac{40}{100} \div \frac{2}{100} = 20$ because $40 \div 2$ is 20.

2. Rewrite the decimals as whole numbers, either by thinking about them as fractions as above, or by moving the decimal point in each number the same number of places, inserting zeros as necessary. As an example, $0.4 \div 0.02$ would require two moves of the decimal point for each number to become a whole number: $40 \div 2$.

Then divide by the usual whole number division algorithm. The quotient will be the same as the quotient of the original decimal numbers.

This algorithm works because the multiplication relationship (the *ratio*) between two numbers stays the same when they are multiplied by a common factor. In the example above, we are multiplying 0.4 and 0.02 each by 100 to obtain 40 and 2.

3. Fractions written in lowest terms with denominators that evenly divide a power of 10 will have terminating decimal representations. Examples include 4 (which divides 100 evenly), 5 (which divides 10), 8 (which divides 1,000 equally). In general, any fraction, which, in its lowest terms, has a denominator with a prime factorization including only 2's and/or 5's will have a terminating decimal representation.

Fractions written in lowest terms with denominators that do not evenly divide any power of 10 will have repeating decimal representations. Examples include 3 (which divides 9, but not 10; 99 but not 100; 999 but not 1,000, etc.), 7 (which divides itself but not 10, 98 but not 100, 994 but not 1000, and 9,996 but not 10,000, etc.) and 15. In general, any fraction, which, in its lowest terms, has a denominator with a prime factorization including any number other than 2 and/or 5 will have a repeating decimal representation.