Investigation 2

ACE Assignment Choices

Differentiated Instruction Solutions for All Learners

Problem 2.1

Core 1–3, 34–39
Other *Applications* 4–6; *Extensions* 46–48

Problem 2.2

Core 7–10, 12–16
Other *Applications* 11; unassigned choices from previous problems

Problem 2.3

Core 17–24, 40, 44
Other *Applications* 20, 21; *Connections* 41–43; *Extensions* 49; unassigned choices from previous problems

Problem 2.4

Core 25, 27–31, 33, 45
Other *Applications* 26, 32; *Extensions* 50–55; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 33 and other ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 34–39: *Bits and Pieces II;* 40–45: *Covering and Surrounding*

Applications

1.–12. There is more than one way to estimate the products. Some possible answers:

1. Rounding numbers to 3×15, we get 45.

2. Rounding numbers to 0.5×120, we find half of 120, which is 60.

3. 0.93 is almost 1. So, the result will be about 12.

4. Considering half of 18, we find 9.

5. Considering one and a quarter of 8, we find 10.

6. Think of 0.8×0.3 instead. Since $8 \times 3 = 24$, the estimate would be 0.24.

7. Estimate: 0.5 ($\frac{1}{2} \times 1 = \frac{1}{2}$)
Exact result: 0.48 ($\frac{6}{10} \times \frac{8}{10} = \frac{48}{100} = 0.48$)

8. Estimate: 3 ($2 \times \frac{3}{2} = 3$)
Exact result: 3.045 ($\frac{21}{10} \times \frac{145}{100} = \frac{3,045}{1,000} = 3.045$)

9. Estimate: 20 ($4 \times 5 = 20$)
Exact result: 19.8744 ($\frac{3,822}{1,000} \times \frac{52}{10} = \frac{198,744}{10,000} =$ 19,8744)

10. Estimate: 1.3 ($1 \times 1.3 = 1.3$)
Exact result: 1.1745 ($\frac{9}{10} \times \frac{1,305}{1,000} = \frac{11,745}{10,000} =$ 1.1745)

11. Estimate: 15 ($5 \times 3 = 15$)
Exact result: 14.877 ($\frac{513}{100} \times \frac{29}{10} = \frac{14,877}{1,000} =$ 14.877)

12. Estimate: 28 ($4 \times 7 = 28$)
Exact result: 28.0224 ($\frac{417}{100} \times \frac{672}{100} = \frac{280,224}{10,000} =$ 28.0224)

13. $0.42 \times \$2.95 = \$1.239$ (you will be charged \$1.24)

14. **a.** 48.98 m^2
b. \$288.98

15. A

16. **a.** 5 **b.** 0.5 **c.** 0.05

17. N = 0.3 **18.** N = 0.06 **19.** N = 3

20. F

21. **a.** 0.3 of his whole garden is planted in early corn. 0.1 of his whole garden is planted in late corn.
b. 2.4 acres of early corn and 0.8 acres of late corn.

22. In estimating 0.52×18.3, 0.52 is about 0.5, which as a fraction is the same as $\frac{1}{2}$. And, 18.3 can be rounded to 18. So, $\frac{1}{2}$ of 18 is 9.
For 1.262×7.94, I can round the numbers to 1.25×8. Then, 1.25 as a fraction is the same as $1\frac{1}{4}$. Since $\frac{4}{4}$ of 8 is 8, and $\frac{1}{4}$ of 8 is 2, the estimate is $8 + 2$ or 10.

23. Ali is right. If two positive numbers are both less than 1, then their product is always less than 1. Ahmed's mistake is to put in only one decimal place. We need to take into account both decimal places of 0.8 and 0.3. Since each number has one decimal place, the result will have $1 + 1 = 2$ decimal places. So, the correct estimation is 0.24.

24.a. Greater than 0.153. This is because 3.4 is more than 1 and we can consider this product as saying "*take a little more than 3 copies of 0.153 added to each other,*" which definitely gives more than 0.153 itself.

b. Less than 3.4. This is because 0.153 is less than 1. If we imagine the fractional equivalent of 0.153, which will be less than a whole, the product is less than a whole of the number 3.4.

25. a. Less than 57.132. Consider the product 0.682×57.132. Since 0.682 is less than 1, by the same reasoning as in 24b, the product will be less than 57.132.

b. Greater than 0.682. By the same reasoning as in 24a, we are looking at more than 57 copies of 0.682 added to each other.

26. a.–b. Both numbers are less than 1, so, using the same idea as in 24b, we can say that we are looking at less than a whole of both numbers. As a result, the product is going to be less than both numbers.

27. a. 9.36 **b.** 0.936
c. 0.0936 **d.** 0.936
e. 0.0936 **f.** 0.00936

28. a. 47.27 **b.** 472.7
c. 14.5 **d.** 3260

29. a. 430.5 **b.** 43.05
c. 0.4305 **d.** 12.3
e. 0.35 **f.** 4.305

30. The sum of the number of decimal places in each factor is equal to the number of decimal places in the product. (Note: This is the case when you don't count "extra" zeros. For example, in 3.2×5.30, you have one decimal place in 3.2 and one in 5.3.) The multiplication 3.2×5.3 has one decimal place in each factor for a total of 2 places. The product will need to have two places. Since $32 \times 53 = 1{,}696$, the product of 3.2×5.3 will be 16.96.

31. We can multiply 27 by 463 and insert three decimal places. Because 2.7 is tenths and 4.63 is hundredths, the product will be a decimal with a thousandths in the answer. Thus, from $27 \times 463 = 12{,}501$, we see that 2.7×4.63 must be 12.501.

32. a. 13.2 **b.** 132
c. 1,320 **d.** 13,200
e. 124.5 **f.** 1,245
g. 12,450 **h.** 124,500

33. a. "Add a zero," in this context, means "place a zero on the right side of number." So, we put a zero at the end of 20 and get 200. The zero is holding the decimal in a certain place. In this case the decimal was moved one place to the right by multiplying by 10, and the zero is "*added*" or "*put*" on the end of the 20 to make it 200.

b. He is wrong again, because placing a zero on the right side of a number is done when the multiplication causes the decimal to move enough places that it is necessary to show that the place value is used but it represents 0 of that place. For example, in 20 there are 2 tens and 0 ones. Since 0.02 was multiplied by 10, moving the decimal one to the right makes the product ten times greater. A digit already exists in the place-value spot to the right, so just move the decimal over one place to get 0.2.

c. Each time you multiply by 10 the decimal is moved one place to the right to make the value ten times greater. If you multiply by 100 you move the decimal two places right since 100 is 10×10, or has two tens. Since 1,000 is $10 \times 10 \times 10$ or three tens, the number will be 1,000 times greater and the decimal moves 3 to the right. You only add zeros when there are not enough existing digits to move the decimal where it is needed.

Connections

34. $\frac{28}{27}$ or $1\frac{1}{27}$ **35.** 6

36. $\frac{4}{3}$ or $1\frac{1}{3}$ **37.** $\frac{4}{3}$ or $1\frac{1}{3}$

38. $\frac{32}{3}$ or $10\frac{2}{3}$ **39.** $\frac{69}{20}$ or $3\frac{9}{20}$ or 3.45

40. a. Carpet C is the longest.

b. Carpet B has the greatest area (about 22.56 m^2).

c. Carpet A is \$349.40, Carpet B is \$312.47, Carpet C is \$325.54. Carpet A is the most expensive. Carpet B is the least expensive.

41. 2.082 $in.^2$

42. 153.6416 cm^2

43. 366.44535 ft^2

44. $\approx$ 128.28 m^2

45. a. Possible answer: length = 8 ft and width = 7 ft

b. Possible answers: length = 0.8 ft and width = 7 ft or length = 8 ft and width = 0.7 ft

c. Possible answers: length = 0.8 ft and width = 0.7 ft, length = 8 ft and width = 0.07 ft, or length = 0.08 ft and width = 7ft

Extensions

46. 0.93 **47.** 0.2601 **48.** 22.10425

49. a. $\frac{3.7}{23} \times \frac{10}{10} = \frac{37}{230}$

b. $\frac{1.6}{4} \times \frac{10}{10} = \frac{16}{40}$ or $\frac{2}{5}$

c. $\frac{3.7}{23} \times \frac{1.6}{4} = \frac{37}{230} \times \frac{16}{40}\left(\frac{2}{5}\right) = \frac{592}{9{,}200}\left(\frac{37}{575}\right)$ (or 0.0643478)

50.

$$\begin{array}{r} 15.24 \\ \underline{\times\ 2.9} \\ 13716 \\ \underline{3048} \\ 44.196 \end{array}$$

51. Belinda's answer is closer to the exact answer. Tanisha rounds 5.2 to 5 and loses 0.2 × 100.4 which is more than 20. Belinda rounds 100.4 to 100 and loses 0.4 × 5.2 which is much less than what Tanisha loses.

52. When multiplying two decimal numbers, we multiply the two numbers as if they do not have any decimal points. Only at the end of multiplication, we count the total number of decimal places in the factors and locate the decimal point in the result accordingly. (Note: This is analogous to the fact that we do not need common denominators to multiply common fractions, while we do to add them.)

53. a. 2.25 m^2

b. 0.25 m^2, 0.5 m^2, 0.5 m^2, and 1 m^2

c. The sum of the areas of part (b) is equal to the area of part (a): $0.25 + 0.5 + 0.5 + 1 = 2.25$.

54. a.–b. Three possibilities: 0.487 × 51.2 or 4.87 × 5.12 or 48.7 × 0.512

Note that the total number of decimal places in the factors must be 4 for each possibility.

55. a. −150 points

b. −250 points

c. She needs +100 points.

d. Answers will vary. Possible answer: She got a 200-point question right and a 50-point question wrong.

Possible Answers to Mathematical Reflections

1. Write each decimal as a fraction with a power of 10 (10, 100, 1,000, etc.) in the denominator. If the number is greater than 1, write it as an improper fraction. Multiply the numerators of these fractions to get the numerator of the product. Multiply the denominators of these fractions to get the denominator of the product. Write the product as a decimal. For example, 0.6 × 0.12 is equivalent to $\frac{6}{10} \times \frac{12}{100} = \frac{72}{1{,}000}$. You can rename the product $\frac{72}{1{,}000}$ as a decimal or 0.072.
2. One strategy is to round one or both decimal numbers to the nearest whole number and multiply those values. For example, 3.9 × 4.098 is approximately 16 because 4 × 4 = 16. Another strategy involves benchmarks. For example, in the problem 0.49 × 12.3, 0.49 can be approximated with the benchmark $\frac{1}{2}$ and 12.3 as 12. $\frac{1}{2}$ of 12 is 6.

ACE ANSWERS 2