## Investigation 3

## ACE <br> Assignment Choices <br> Differentiated Instruction

## Problem 3.1

Core 1-4
Other Connections 21-27, Extensions 43

## Problem 3.

Core 5-8
Other Applications 9-12, Connections 28-37, Extensions 44-47; unassigned choices from previous problems

## Problem 3.3

Core 13-19
Other Applications 20; Connections 38-42; Extensions 48, 49; unassigned choices from previous problems
Adapted For suggestions about adapting ACE exercises, see the CMP Special Needs Handbook.
Connecting to Prior Units Units 21, 23, 26, 28-32, 37: Covering and Surrounding; 22, 33: Data About Us; 24, 35: Shapes and Designs; 25, 27, 36: Prime Time; 38, 39: Bits and Pieces II

## Applications

1. a. Basketball Road Trip

| Time (hr) | Distance $(\mathrm{mi})$ |
| :---: | :---: |
| 0 | 0 |
| 1 | 60 |
| 2 | 120 |
| 3 | 180 |
| 4 | 240 |
| 5 | 300 |
| 6 | 360 |
| 7 | 420 |
| 8 | 480 |
| 9 | 540 |
| 10 | 600 |


b. Estimates should be close to the following: $120 \mathrm{mi}, 165 \mathrm{mi}, 210 \mathrm{mi}, 435 \mathrm{mi}$
c. The data are represented in the table by corresponding $x$ and $y$ values 2 and 120 . They are represented by point $(2,120)$ on the graph.
d. These values are not in the table. They would be the values $\frac{3}{4}$ of the way between $(2,120)$ and $(3,180)$. If the plotted points on the graph are connected, then the values are represented by the point $(2.75,165)$ on the line.
e. The distance (in miles) is 60 multiplied by the time (in hours).
f. $1 \frac{1}{2}$ hours
g. about 3 hours 7 minutes; $\frac{1}{3}$ of 560 mi is about 187 mi
h. $9 \frac{1}{3}$ hours, or 9 hours 20 minutes
2. a.

Bike Ride

| Time (hr) | Distance (mi) |
| :---: | :---: |
| 0 | 0 |
| 0.5 | 4 |
| 1.0 | 8 |
| 1.5 | 12 |
| 2.0 | 16 |
| 2.5 | 20 |
| 3.0 | 24 |
| 3.5 | 28 |
| 4.0 | 32 |
| 4.5 | 36 |
| 5.0 | 40 |

b. 8 miles; 48 miles; 68 miles; 80 miles
3. a. Traveling at 70 mph

| Time (hr) | Distance (mi) |
| :---: | :---: |
| 0 | 0 |
| 0.5 | 35 |
| 1.0 | 70 |
| 1.5 | 105 |
| 2.0 | 140 |
| 2.5 | 175 |
| 3.0 | 210 |
| 3.5 | 245 |
| 4.0 | 280 |

b.

c. $d=175$ miles
d. $t=3$ hours
e. Yes, in this case the information represented by a line connecting the points is accurate because the distance increases at a constant rate (so there would not be any jumps or curves between points).
4. a. $L=4.25 n$
b. $L=4.25 \times 25=\$ 106.25$
c. We need to find $n$ so that $\$ 89.25=4.25 n$. What number multiplied by 4.25 gives $89.25 ? n=21$ people
5.

| $x$ | 1 | 2 | 5 | 10 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 7 | 11 | 23 | 43 | 83 |

6. 

| $k$ | 1 | 2 | 5 | 10 | 20 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $m$ | 99 | 98 | 95 | 90 | 80 |

7. 

| $t$ | 1 | 2 | 5 | 10 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $d$ | 3.5 | 7 | 17.5 | 35 | 70 |

8. a. $\$ 290$ after 1 payment; $\$ 265$ after 2 payments; $\$ 240$ after 3 payments
b. $a=315-25 n$
C. Sean's Loan Payments

| Number <br> of Months | Amount <br> Owed |
| :---: | :---: |
| 0 | $\$ 315$ |
| 1 | $\$ 290$ |
| 2 | $\$ 265$ |
| 3 | $\$ 240$ |
| 4 | $\$ 215$ |
| 5 | $\$ 190$ |
| 6 | $\$ 165$ |
| 7 | $\$ 140$ |
| 8 | $\$ 115$ |
| 9 | $\$ 90$ |
| 10 | $\$ 65$ |
| 11 | $\$ 40$ |
| 12 | $\$ 15$ |
| 13 | $-\$ 10$ |

## Sean's Loan Payments


d. As $n$ increases by $1, a$ decreases by $\$ 25$. As you read down the columns of the table, the amount owed decreases by $\$ 25$ each time the number of months increases by 1 . The points of the graph lie on a line that slants downward as you read from left to right. More specifically, if we move 1 unit to the right on the $x$-axis, the line moves downward 25 units on the $y$-axis.
e. Sean will pay off the loan in 13 months. His last payment will be only $\$ 15$. In the table, the amount owed drops below 0 on month 13. In the graph, the point for $x=13$ is below the $x$-axis.
9. $A=l w$, where $A$ is the area, $l$ is the length, and $w$ is the width
10. $h=2 s$, where $h$ is the number of hot dogs and $s$ is the number of students
11. $y=4 w$, where $y$ is the amount of material in yards and $w$ is the number of windows
12. $t=2+1.10 \mathrm{~m}$, where $t$ is the taxi fare in dollars and $m$ is the number of miles
13. $t=0.08 p$
14. $d=550 h$
15. $c=0.25 p$
16. $b=0.05 m+49$
17. $y$ is 4 times $x ; y=4 x$
18. $t$ is 50 minus $s ; t=50-s$
19. $z$ is 1 more than 5 times $n ; z=5 n+1$
20. D

## Connections

21. a.

| Side (units) | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: |
| Perimeter (units ${ }^{2}$ ) | 4 | 8 | 12 | 16 | 20 | 24 |

The perimeter increases by 4 as the length of the side increases by 1 .
b.

| Side (units) | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: |
| Area (units ${ }^{2}$ ) | 1 | 4 | 9 | 16 | 25 | 36 |

The area is the square of the length of the side. It grows at an increasing rate as side length increases.
22. $\frac{(4.5+7.3)}{2}=5.9$
23. $A=\pi 6^{2}=36 \pi \mathrm{~cm}^{2} \approx 113.04$
24. Triangle: $180^{\circ}$; parallelogram: $360^{\circ}$; pentagon: $540^{\circ}$; hexagon: $720^{\circ}$
25. 19
26. $A=\frac{1}{2} \times 10 \times 15=75 \mathrm{~cm}^{2}$
27. 4,725
28. a. Kai will travel 84.78 in. in one turn because the circumference of his wheel is about $3.14 \times 27 \mathrm{in}$., or about 84.78 in .
b. Masako will travel about 62.8 in. for one turn because the circumference of her wheel is about $3.14 \times 20 \mathrm{in}$., or 62.8 in .
c. Kai will travel about 42,390 in. for 500 turns because $84.78 \times 500=42,390$. Students may convert this to $3,532.5 \mathrm{ft}$ or $1,177.5 \mathrm{yd}$.
d. Masako will travel about $31,400 \mathrm{in}$. for 500 turns because $62.8 \times 500=31,400$. Students may convert this to $2,617 \mathrm{ft}$ or 872 yd.
e. $1,200 \div 84.78 \approx 14.15$ turns
f. $1,200 \div 62.8 \approx 19.12$ turns; 1 mi is $5,280 \mathrm{ft}$ or $63,360 \mathrm{in}$. So, it would take $63,360 \div 62.8 \approx 1,008.92$ turns to cover 1 mi .
29. a. 2.5 ft
b. The bike will travel about $3.14 \times 5=15.7 \mathrm{ft}$ in one turn, so in 100 turns it will travel about $1,570 \mathrm{ft}$.
c. Because $3 \mathrm{mi}=15,840 \mathrm{ft}$, it will make $15,840 \div 15.7 \approx 1,008.9$ turns.
d. The big wheel can go three times as far for the same number of turns because its diameter is three times the diameter of Masako's wheel.
30. $A=l w$
31. $A=b h$
32. $P=2 b+2 h$ or $P=2(b+h)$
33. $m=\frac{(p+q)}{2}$
34. $A=\pi r^{2}$
35. $S=(n-2) 180$
36. $O=2 n-1$
37. $A=\frac{1}{2} b h$
38.

| $x$ | $\frac{1}{5}$ | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{2}{5}$ | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{3}{4}$ | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $\frac{7}{10}$ | $\frac{3}{4}$ | $\frac{5}{6}$ | $\frac{9}{10}$ | 1 | $\frac{7}{6}$ | $\frac{5}{4}$ | $\frac{11}{2}$ |

39. 

| $x$ | $\frac{1}{5}$ | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{2}{5}$ | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{3}{4}$ | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $\frac{1}{10}$ | $\frac{1}{8}$ | $\frac{1}{6}$ | $\frac{1}{5}$ | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{3}{8}$ | $\frac{5}{2}$ |

40. $y$ is equal to $x$
41. $y$ is 1 more than $x$
42. $y$ is 5 minus $x$

Extensions
43. a.

Speed for a Car Trip

| Distance <br> $(\mathrm{mi})$ | Time <br> $(\mathrm{hr})$ | Average Speed <br> $(\mathrm{mi} / \mathrm{h})$ |
| :---: | :---: | :---: |
| 145 | 2 | 72.5 |
| 110 | 2 | 55 |
| 165 | 2.5 | 66 |
| 300 | 5.25 | $\approx 57.1$ |
| 446 | 6.75 | $\approx 66.1$ |
| 528 | 8 | 66 |
| 862 | 9.5 | $\approx 90.7$ |
| 723 | 10 | 72.3 |

b. $s=\frac{d}{t}$
44. 45 club members
45. a. 11 rides
b. 14 rides
46. 50 seconds, 160 seconds
47. 90 seconds, 120 seconds
48. a. (Figure 3)
b. $t=3+4(b-1)$, or $t=4 b-1$; There are many possible explanations. One way to look at this pattern is that a bridge with 1 toothpick on the bottom requires 3 toothpicks, and each additional bottom toothpick requires 4 additional toothpicks.
c. The triangular design provides rigidity that the design made of squares doesn't. This is a principle that was developed in Shapes and Designs of Course 1.
49. Students need to subtract the cost of the amusement park from the tour profit with van found in Question $D(2)$ of Problem 3.3. A way to approach this is to make a table using

Figure 3
Rod Bridges

| Rods Along Bottom | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total Number of Rods | 3 | 7 | 11 | 15 | 19 | 23 | 27 | 31 | 35 | 39 |

columns 1 and 6 from the table made for Question A of Problem 3.3 and add two new columns. (Figure 4)
Reasoning symbolically:
Cost of Amusement park $=50+10 n$;
Tour profit with van $=350 n-155 n-700$;
Tour profit with amusement park =
$350 n-155 n-700-(50+10 n)=185 n-750$
This approach is more sophisticated and requires students to think about representing the subtraction of a quantity $(50+10 n)$ from another quantity $350 n-155 n-700$.

Leaving the result as:
Tour profit with amusement park $=$ $350 n-155 n-700-(50+10 n)$ is fine at this point. In Moving Straight Ahead and Say It With Symbols, students will work more with writing and simplifying equations of this form.

## Possible Answers to Mathematical Reflections

1. First, identify the variables involved in the relationship. Then, choose letters to stand for those variables. Deciding which variable is
the dependent variable and which is the independent variable is helpful. Calculate some values of the dependent variable for specific values of the independent variable, and look for a pattern in those calculations. Describe the pattern in words. Finally, use the words as a guide to write an equation to represent the relationships.
2. Equations give you a short way to represent a relationship. They make calculating specific values more precise and efficient.
3. Tables allow you to see pairs of values. However, the number of pairs you can show in a table is limited. Also, it can be difficult to see the overall pattern in a table.
Graphs make it easy to see the overall pattern visually. However, it is harder to find exact values from a graph.
Equations provide a brief summary of the relationship and allow you to find values of one variable for any value of the other variable. Although you can tell some things about the overall pattern from an equation, it doesn't give a visual picture of the pattern.

Figure $4 \quad$ Tour Revenue and Expenses

| Number of <br> Customers | Profit | Tour Profit <br> With Van | Cost of <br> Amusement <br> Park | Profit With Van <br> Minus Cost of <br> Amusement Park |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $\$ 195$ | $-\$ 505$ | $\$ 60$ | $-\$ 565$ |
| 2 | $\$ 390$ | $-\$ 310$ | $\$ 70$ | $-\$ 380$ |
| 3 | $\$ 585$ | $-\$ 115$ | $\$ 80$ | $-\$ 195$ |
| 4 | $\$ 780$ | $\$ 80$ | $\$ 90$ | $-\$ 10$ |
| 5 | $\$ 975$ | $\$ 275$ | $\$ 100$ | $\$ 175$ |
| 6 | $\$ 1,170$ | $\$ 470$ | $\$ 110$ | $\$ 360$ |
| $n$ |  | $350 n-155 n-700$ | $50+10 n$ | $185 n-750$ |
|  |  |  |  | $=350 n-155 n-$ <br> $700-(50+10 n)$ |

