Investigation 1

ACE Assignment Choices

Differentiated Instruction Solutions for All Learners

Problem 1.1

Core None
Other *Connections* 13–17

Problem 1.2

Core 1–3
Other *Connections* 18–20; unassigned choices from earlier problems

Problem 1.3

Core 4, 5, 26
Other unassigned choices from earlier problems

Problem 1.4

Core 6–9, 27
Other *Connections* 21–24; *Extensions* 28; unassigned choices from earlier problems

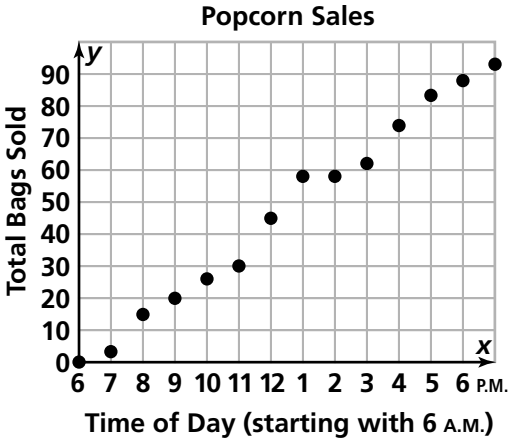
Problem 1.5

Core 10, 12
Other *Applications* 11, *Connections* 25; *Extensions* 29, 30; unassigned choices from earlier problems

Adapted For suggestions about adapting Exercise 10 and other ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 13–15: *Bits and Pieces I*; 18–20: *How Likely Is It?*; 21–24: *Data About Us*

Applications

1. a. Time is on the *x*-axis because the number of bags sold depends on the time.



b. Answers will vary. It is important to give students examples of complete and thoughtful responses early in this unit so they have a sense of what is expected. Possible response: Very few bags were sold before 7:00 A.M., perhaps because most people do not eat popcorn so early in the morning. The number jumped by 12 bags between 7:00 A.M. and 8:00 A.M., when people may have stopped for a snack on their way to work or school. The number went up by about 5 bags an hour between 8:00 A.M. and 11:00 A.M. Sales rose by 15 bags between 11:00 A.M. until noon and by 13 bags from noon to 1:00 P.M, when people may have been buying lunch. No popcorn was sold from 1:00 P.M. to 2:00 P.M., and only 4 bags were sold between 2:00 P.M. and 3:00 P.M. Then, the number jumps again by 12 bags from 3:00 P.M. to 4:00 P.M. Maybe people were buying a midafternoon snack. During the next 3 hours, between 4:00 P.M. and 7:00 P.M., the number sold drops from 9 bags to 5 bags to 4 bags. Dinner time is probably the cause of this decrease in sales.

c. 11 A.M. to noon; 1 P.M. to 2 P.M.

2. a. Possible answer: The graph shows that Mary was timed for 120 seconds. It shows the number of jumping jacks she did in each 10-second increment of the 120-second time period.

b. The graph from Problem 1.2 showed the *total* number of jumping jacks so far at the end of each 10-second interval. Each point on that graph is higher than the one before because the total number increased with each interval, even when the number done *within* each interval decreased. In Mary's graph, the points go up and down because the number of jumping jacks in some intervals is less than in others.

c. 135 jumping jacks

3. Ken did not do more jumping jacks in 120 seconds. Ken's points are higher because the scales on the two graphs are different. The *y*-axis on Andrea's graph goes to 160, while the *y*-axis on Ken's graph goes only to 80. Andrea did about 110 jumping jacks in 120 seconds while Ken did only about 72.

4. a. (Figure 4)

b. between birth and age 1 (9 inches)

c. from age 14 to 16 and from age 17 to 18

d. It makes sense to connect the points because growth occurs between birthdays. (The question of how these points should be connected, by line segments or a curve, is another point of discussion.)

e. Answers will vary. The exact change in height is easier to read from the table. However, students may argue that the graph provides a better overall picture.

5. a. 6 hours after midnight, or 6:00 A.M.; 16.2 m

b. noon; 10.0 m.

c. The water depth changes most rapidly—by 1.7 meters—from 2 to 3 (2 A.M.–3 A.M.), from 8 to 9 (8 A.M.–9 A.M.), and from 14 to 15 (2 P.M.–3 P.M.).

d. (Figure 5) The graph has two humps. It looks symmetric, so that if it were flipped over x = 12 (hour 12), the two parts would align. The graph rises until hour 6, falls until hour 12, and rises again to hour 18, and then falls again.

Figure 4

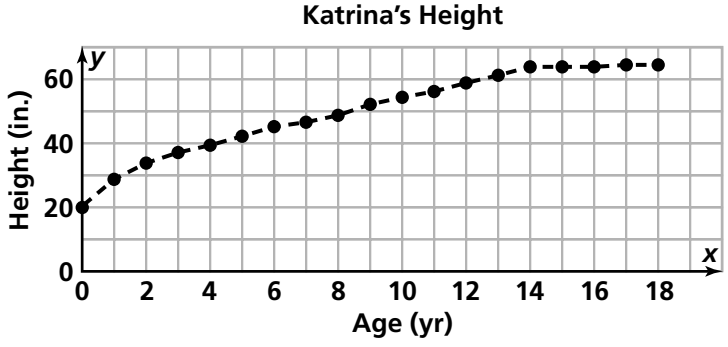
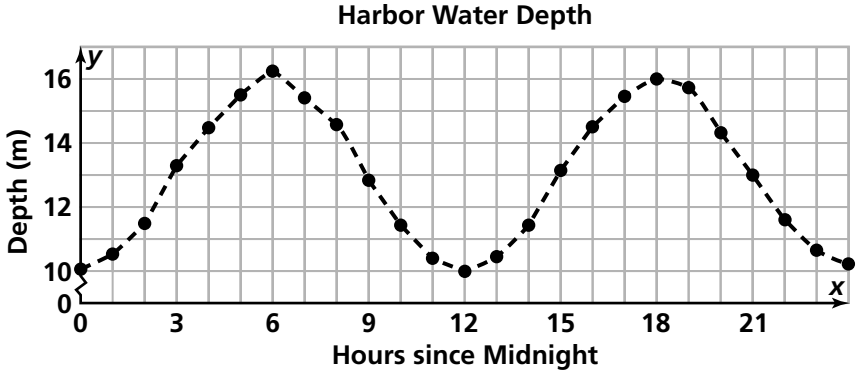


Figure 5



e. Possible answer: I used 1-hour intervals on the x-axis because that was the time interval in the table. I used 2-meter intervals on the y-axis because it allowed all the data to be graphed on my grid paper. (Not all students will use this scale.)

6. a. In all three graphs the population increases at a steady rate over time (by about 500 people per year).

b. Yes. All three graphs represent the same data. However, the y-axis scales of the graphs are different, giving different impressions of how fast the growth was.

7. a. The two variables are time of day and the number of cans sold each hour.

b. (NOTE: Students may be confused because this is not a cumulative graph. Unlike their graphs showing the jumping jack data, each point in this graph tells how many cans were sold during the one hour preceding that time.) Possible answer: Very few cans were sold before 7:00 A.M., probably because not many people were at school so early. The number jumps to 80 cans at 8:00 A.M. People may have bought juice when they arrived at school. The number drops at 9:00 A.M., when people may have been in class. The number jumps to about 100 cans by 10:00 A.M., when people may have taken a mid-morning break. The number drops to around 20 cans at 11:00 A.M. At noon it jumps to about 180 cans, when some people may have bought juice to go with their lunches. The number goes down again; at 2:00 P.M. about 100 cans were sold. Then the number jumps to about 160 cans at 3:00 P.M., when classes may have been over. The number of cans sold decreases until 6:00 P.M., when no cans were sold. Perhaps most people had already left the school. The number peaks again at 8:00 P.M. when about 120 cans were sold and drops off again until 10:00 P.M. when only about 10 cans were sold. Maybe there was some after-school activity that brought people to school at 8:00 P.M. and then the building closed at 10:00 P.M.

8. a. The variables are time (in hours) and temperature (in degrees Fahrenheit).

b. Values may vary slightly.

Temperatures for Day 1

Time (hr)	Temperature (°F)
0	60
0.5	52
1	57
1.5	60
2	70
2.5	80
3	70
3.5	65
4	70
4.5	80
5	85

c. Possible answer: The high temperature was around 85°F and the low temperature was around 52°F, so the difference is 85 – 52, or 33°F. (Any answer between 31°F and 36°F is acceptable.)

d. The temperature rose fastest between hours 1.5 and 2, 2 and 2.5, and 4 and 4.5. It fell the fastest between hours 2.5 and 3.

e. Answers will vary, but generally it is easier to find the exact size of an increase or decrease using a table.

f. Answers will vary, but generally it is easier to use a graph to find the interval of greatest change because exact calculations do not need to be made.

g. Connecting the points shows the temperature changing at a steady rate between half-hour marks. It makes sense to connect the points because time is a continuous variable, so we will have temperature after 15 minutes, after 37 minutes, and so on. The information may not be completely accurate because the temperature may not have changed at a constant rate. However, it is useful for making estimates.

9. a. A constant speed over a period of time

b. Possible answer: The graph is not reasonable for a cyclist or for the wind under normal conditions. A rider's speed can be affected by fatigue or environmental factors such as

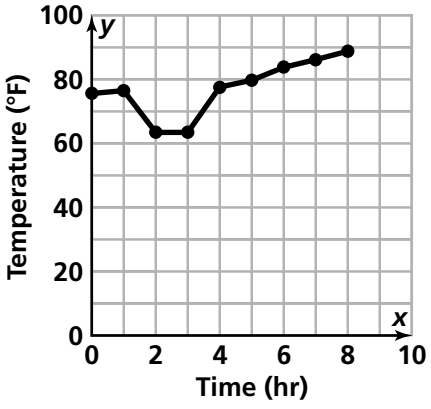
temperature, wind speed or direction, and terrain. A van could travel close to a constant speed on a flat surface. The wind usually comes in gusts. It does not seem that it would remain constant over a long period of time. However, one thoughtful student answered: "We don't know what the scale is. So if a small amount of space on the *y*-axis means millions and millions, then this graph is possible for the rider, the van, or the wind because their small amount of speeding up and slowing down wouldn't show up on the graph."

10. Answers will vary, but the graph and the table should show that it was warm at 8 A.M. (at time = 0 hr). Then, the temperature decreased rapidly to 63°F by midmorning and stayed constant for about an hour. After this, the temperature increased until it reached 89°F at 4 P.M.

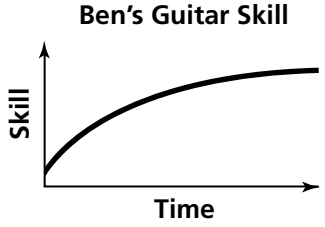
A Day on the Road

Time (hr)	Temperature (°F)
0	76
1	77
2	63
3	63
4	78
5	80
6	83
7	86
8	89

A Day on the Road



11. a. A possible graph is shown.



b. The variables are time and skill.

c. Possible answer: hours of practice and the frequency of his lessons

12. Graph I shows Amanda's hunger and Graph II shows her happiness. The increases are quite gradual with hunger and the decreases are rather sudden when Amanda eats. The graph for Amanda's happiness shows that she can stay at the same level of happiness for a while, such as when she is having fun at basketball practice from 4 to 6.

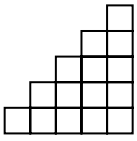
Connections

13. 0.25, 0.5, 0.75, 1.00, 1.25, 1.5, 1.75, 2.0; add 0.25

14. $\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}, 1$; add $\frac{1}{8}$.

15. $\frac{1}{6}, \frac{1}{3}, \frac{4}{6}, \frac{4}{3}, \frac{8}{3}, \frac{32}{6}$; multiply by 2

16.



Number of Squares in Bottom Row	1	2	3	4	5
Total Number of Squares	1	3	6	10	15

17.

Width of Base	3	5	7
Total Number of Cubes	10	35	84

18. A

19. a. 2 out of 6, or $\frac{2}{6}$ (or 1 out of 3, or $\frac{1}{3}$)

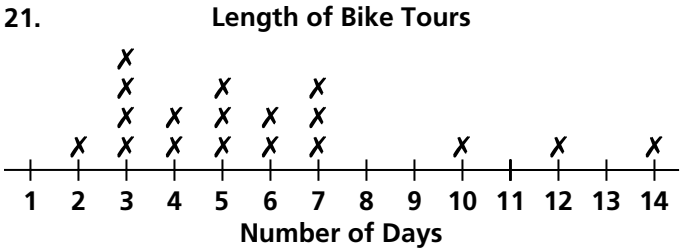
b. Your probability of winning increases each time you add another winner block. If you have only 1 winner block, your probability

of winning is $\frac{1}{5}$. However, if you have 6 winner blocks (10 blocks total), your probability of winning is $\frac{3}{5}$.

Number of Winners	1	2	3	4	5	6
Total Number of Blocks	5	6	7	8	9	10
Probability of Winning	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{3}{7}$	$\frac{1}{2}$	$\frac{5}{9}$	$\frac{3}{5}$

20. 3 out of 36 ($\frac{3}{36}$), or 1 out of 12 ($\frac{1}{12}$)

21.



22. G

23. Answers will vary. Some students may note that the 3-day tour is the most preferred length (that is, 3 days is the mode) and surmise that a 3-day trip is the best option. However, other students may observe that half of the most popular tours are shorter than 5 days (using the median), and half are longer than 5 days, so a 5-day tour is the average length and would be a popular option.

24. a. (Figure 6)

b. The greatest increases occur after 12 and 16 years of education. This is probably because a diploma qualifies a person for higher-paying jobs. (You may want to point out to students that these are not starting salaries. Some of these people have been in their field for a number of years. The participants of this study are people over 25.)

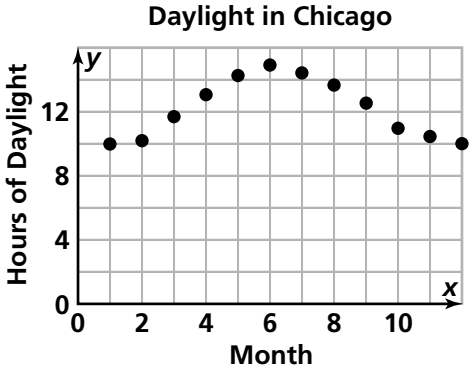
c. Answers will vary. It is often easier to see changes, or jumps, in a graph, but it is easier to use a table to find the exact amount of those changes. As the months increase by 1, the daylight hours increase until month 6 and then decrease.

25. Answers will vary.

Extensions

26. a. Possible answer: The hours of daylight are least in Jan. and Dec. They are greatest in Jun. The hours of light change most rapidly (by 1.5 hr) from Feb. to Mar. and Sep. to Oct.

b.



Possible answer: The number of hours of light increases slowly from Jan. to Jun. and reaches a peak at Jun. It decreases slowly from Jun. to Dec.

c.

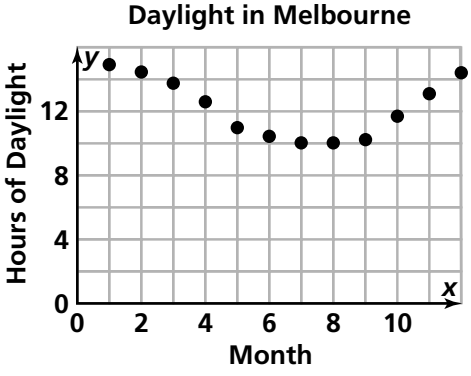


Figure 6

Years of Education	8	9	10	11	12	13	14	15	16
Median Salary	$12,500	$14,000	$16,500	$17,500	$28,000	$30,500	$34,000	$36,000	$49,000

d. (Figure 7)

27. a. 10 jumping jacks; 20 jumping jacks; 40 jumping jacks

b. Answers will vary.

c. At the end of 30 seconds, the jumper would have done between 25 and 30 jumping jacks. At the end of 40 seconds, the jumper would have done between 30 and 35 jumping jacks. At the end of 50 seconds, the jumper would have done between 35 and 40 jumping jacks.

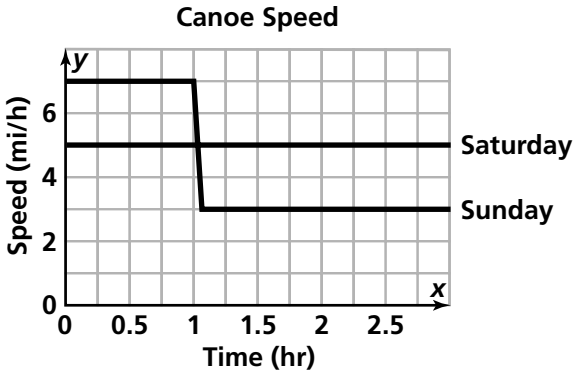
d. There was a constant increase in the number of jumping jacks for the first 25 seconds. Then, the jumping jack rate started to decrease.

e. It represents the overall trend that as the number of seconds increases, the number of jumping jacks increases. However, it does not show specific details, such as the rate of increase slowing down a bit.

28. a. Answers will vary. Students might make a reasonable argument for any of the graphs. Yet, some graphs seem to be better than others. The following arguments assume that the intersection of the *x*- and *y*-axes is point (0, 0) on all graphs. Unlike graph i, graphs ii, iii, and iv represent the idea that profit will not go up indefinitely. When you raise the price too high, some customers will stop buying. Graphs iii and iv show that there is a price that results in the maximum profit. Graph iv is a better representation because graph iii shows the unlikely event of making a profit at a very low price for each shirt. Students might draw a more detailed graph that shows a negative profit (loss) when the price is too low.

b. Possible answers: Selling price, price the club must pay for the sweatshirts, the location and times the booster club chooses for selling sweatshirts, and customer demand (which might depend on other variables such as income and weather).

29. a–b.



c. When Chelsea and Nicole are paddling with the current, their relative speed is 5 mph, the speed they can paddle in calm water, plus the speed of the current. When Chelsea and Nicole are paddling against the current, their relative speed is 5 mph minus the speed of the current.

30. Answers will vary. Possible answers:

a. As distance increases, the time increases.

Distance from School to Home (mi)	Time to Walk Home (min)
$\frac{1}{4}$	5
$\frac{1}{2}$	10
$\frac{3}{4}$	15
1	20
$1\frac{1}{2}$	30

Figure 7

Daylight in Melbourne

Month	1	2	3	4	5	6	7	8	9	10	11	12
Daylight Hours	15	14.5	13.8	12.5	11	10.5	10	10	10.2	11.7	13.1	14.3

b. As the price of popcorn goes up, the number of bags sold goes down.

Price of Popcorn at Theater	Number of Bags Sold
$2	50
$4	40
$6	30
$8	20
$10	10

c. As the airplane speed increases, the time to complete the trip decreases.

Speed of Airplane (miles per hour)	Time to Complete 500-Mile Trip
100	5
125	4
150	3.33
175	2.86
200	2.5

d. As the number of days increases, the late fee increases.

Number of Days	Late Fee
1	$2
2	$4
3	$6
4	$8
5	$10

e. As the length of the call goes up, the charge goes up (except on cell phones, or other special plans).

Length (min)	Cost
1	$0.30
5	$1.50
10	$3.00
15	$4.50
20	$6.00

Possible Answers to Mathematical Reflections

1. **Step 1:** Select two variables you want to represent.

   Step 2: Select an axis to represent each variable. Put the independent variable on the x-axis (horizontal axis). If time is one of the variables, you should put it on the x-axis.

   Step 3: Select a scale for each axis. For each axis, you need to determine the greatest and least values you want to show on your graph and how you want to space the scale marks.

   Step 4: Label your graph so that someone else would know what the graph represents. Give the graph a title.

   Step 5: Plot points on your graph.

2. If time is one of the variables, you should usually put it on the x-axis (the horizontal axis). Otherwise, put the independent variable on the x-axis and the dependent variable on the y-axis.

3. **a.** A table gives values that are easy to read. However, it is often hard to see patterns or trends at a glance without doing some calculations.

   b. A graph offers a visual image from which you can quickly see patterns in the relationship. However, it is often more difficult to read exact values from a graph.

   c. A written report gives information that cannot be contained in a graph or table, such as the reasons a certain section of a trip took longer than another section. However, in a written report it is difficult to notice patterns or trends.