Investigation 4

ACE Assignment Choices

Differentiated Instruction
Solutions for All Learners

Problem 4.1

Core 1–3, 25
Other *Connections* 26, *Extensions* 34

Problem 4.2

Core 4–7, 12, 13, 14, 27–29
Other *Applications* 8–11, 15–18; *Connections* 30; *Extensions* 35; unassigned choices from previous problems

Problem 4.3

Core 19–22, 32
Other *Applications* 23, 24; *Connections* 33; *Extensions* 36; unassigned choices from previous problems

Adapted For suggestions about adapting Exercises 1–3 and other ACE exercises, see the CMP *Special Needs Handbook*.

Applications

1. The path is $7 \times 5 \times 2 \times 3 \times 4$.
2. The path is $3 \times 4 \times 5 \times 6$.
3. Puzzles will vary.
4. $2 \times 2 \times 3 \times 3$
5. $2 \times 2 \times 3 \times 3 \times 5$
6. $3 \times 5 \times 5 \times 7$
7. $3 \times 5 \times 11$
8. 293
9. $2 \times 2 \times 2 \times 5 \times 19$
10. $2 \times 2 \times 2 \times 3 \times 3 \times 3$
11. $3 \times 7 \times 11$
12. $36 = 2^2 \times 3^2$, $180 = 2^2 \times 3^2 \times 5$, $525 = 3 \times 5^2 \times 7$, $165 = 3 \times 5 \times 11$, $293 = 293$, $760 = 2^3 \times 5 \times 19$, $216 = 2^3 \times 3^3$, and $231 = 3 \times 7 \times 11$
13. $2 \cdot 2 \cdot 2 \cdot 3 \cdot 13$
14. D
15. Jamahl is correct. Possible answer: Consider 216. It has six prime factors: $2 \times 2 \times 2 \times 3 \times 3 \times 3$. Then consider 231. It has three prime factors: $3 \times 7 \times 11$. 231 is greater than 216, even though it has fewer prime factors.
16. 10, 20, 30, 40, 50, 60, 70, 80, and 90. The numbers are the multiples of 10 (2×5) less than 100.
17. H
18. $2 \times 3 \times 5 = 30$, $2 \times 3 \times 7 = 42$, $2 \times 3 \times 11 = 66$, $2 \times 3 \times 13 = 78$, $2 \times 5 \times 7 = 70$
19. GCF = 9, LCM = 180
20. GCF = 15, LCM = 150
21. GCF = 26, LCM = 312
22. GCF = 15, LCM = 60
23. GCF = 1, LCM = 1,440
24. GCF = 1, LCM = 444

Connections

25. He can give each child 3 cookies, and he will have 12 left for himself.
26. $8 \times 8 = 64$, so they have 64 grandchildren. They have 512 great-grandchildren.
27. Rosa is correct because the number 1 is not a prime number. Tyee is correct that his string is a longer string of factors, but it is not a string of prime factors for 30.
28. Mathematicians have determined that it is important for a number to be able to be identified by its longest string of factors. If the number 1 were prime, the prime factorization for a number would have to include 1, and *could* include 1 as a factor any number of times. So a prime factorization would not be the same as the longest string possible without using 1.

29. a. 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, and 99

b. 21, 42, 63, and 84

c. 63

d. 126

30. a. Possible answers: $1994 = 2 \times 997$, $1995 = 3 \times 5 \times 7 \times 19$, $1996 = 2 \times 2 \times 499$, $1997 = 1997$, $1998 = 2 \times 3 \times 3 \times 3 \times 37$, $1999 = 1999$, $2000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$.

b. Answers will vary. For example, 1996 is not square; it is a multiple of 4 and of 998. It is not prime, and it is even.

31. Factor to find that $184 = 8 \times 23$ and $207 = 9 \times 23$. 23 is the only common factor other than 1. Therefore Tomas worked 8 days at \$23 per day and Sharina worked 9 days at \$23 per day.

32. Since the number is a multiple of 2 and 7 (Clue 1), it must be a multiple of 14. The multiples of 14 between 50 and 100 (Clue 2) are

$56 = 2 \times 2 \times 2 \times 7$,
$70 = 2 \times 5 \times 7$,
$84 = 2 \times 2 \times 3 \times 7$, and
$98 = 2 \times 7 \times 7$.

Of these numbers, only 70 is the product of three different prime numbers (Clue 3). The number is 70.

33. The factors of 32 are 1, 2, 4, 8, 16, and 32 (Clue 3). Of these numbers, only 1, 16, and 32 have digits that add to odd numbers (Clue 4). 1 and 16 are square numbers (Clue 1). Of these two numbers, only 16 has 2 in its prime factorization (Clue 2). The number is 16.

Extensions

34. a. 52 weeks, with 1 extra day if it is not a leap year and 2 extra days if it is a leap year.

b. January 8, 15, and 22

c. Saturday

d. Your birthday will fall one day later in the week each year, except when leap day (February 29) falls between your birthdays. In that case, your birthday will be two days later in the week. If your birthday is February 29, your birthday will be five days later in the week every time it occurs.

35. Answers will vary. Sample: If a number is the least common multiple of several prime numbers, its prime factorization will contain only those primes, and no others.

36. The common multiples of 2, 3, 4, 5, and 6 are 60, 120, 180, ... If we add the clue that the box contains fewer than 100 books, the only answer would be 60.

Possible Answers to Mathematical Reflections

1. a. Every number (greater than 1) has a prime factorization.

b. Every number has exactly one prime factorization.

c. 1 cannot be prime because otherwise prime factorizations could not be unique.

2. a. If you write down the prime factorizations of two numbers, the shortest factor string that includes the prime factorizations of both numbers gives the least common multiple of the two numbers. For example, $16 = 2 \times 2 \times 2 \times 2$ and $14 = 2 \times 7$, so the least common multiple of 16 and 14 is $2 \times 2 \times 2 \times 2 \times 7 = 112$.

b. If you write down the prime factorizations of two numbers, the longest factor string the two numbers have in common gives the greatest common factor. For example, $36 = 2 \times 2 \times 3 \times 3$ and $32 = 2 \times 2 \times 2 \times 2 \times 2$. The longest factor string the two factorizations have in common is 2×2, so the greatest common factor of 36 and 32 is 4.

c. Two numbers are relatively prime if they have no factors in common. For example, 125 and 64 are relatively prime because $125 = 5 \times 5 \times 5$ and $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$, so there are no common factors. This means that their only common factor is 1.

3. If you know the greatest common factor of two numbers is 1, then the least common multiple will be the product of the two numbers.