Investigation 3

ACE Assignment Choices

Differentiated Instruction
Solutions for All Learners

Problem 3.1

Core 1–5, 31, 32
Other *Applications* 6–8

Problem 3.2

Core 10, 11, 15
Other *Applications* 9, 12–14; *Connections* 33–35; *Extensions* 37, 38; unassigned choices from previous problems

Problem 3.3

Core 16–21
Other *Applications* 22, 23; *Extensions* 39–41; unassigned choices from previous problems

Problem 3.4

Core 24–28, 30
Other *Applications* 29, *Connections* 36, *Extensions* 42–44; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 28 and other ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 32: elementary school multiplication algorithms

Applications

1. 24, 48, 72, and 96; the least common multiple is 24.
2. 15, 30, 45, 60, 75, and 90; the least common multiple is 15.
3. 77; the least common multiple is 77.
4. 90; the least common multiple is 90.
5. 72; the least common multiple is 72.
6. 100; the least common multiple is 100.
7. 42, 84; the least common multiple is 42.
8. 60; the least common multiple is 60.
9. **a.** Possible answers: 3, 5; 8, 9; 7, 11
   b. They have no common factors except 1.
10. Possible answers: 2, 5; 1, 10
11. Possible answers: 4, 9; 18, 36
12. Possible answers: 4, 15; 12, 5
13. Possible answers: 3, 35; 7, 15
14. **a.** Twenty-four 1-hour shifts; twelve 2-hour shifts; eight 3-hour shifts; six 4-hour shifts; four 6-hour shifts; three 8-hour shifts; two 12-hour shifts, and one 24-hour shift. These are all factors of 24.
    b. 45 seconds, which is the least common multiple of 9 and 15
15. 24 days
16. 1, 2, 3, and 6; the greatest common factor is 6.
17. 1; the greatest common factor is 1.
18. 1, 3, 5, and 15; the greatest common factor is 15.
19. 1 is the only common factor.
20. 1, 7; the greatest common factor is 7.
21. 1, 5; the greatest common factor is 5.
22. 1, 2; the greatest common factor is 2.
23. 1, 3, 7, 21; the greatest common factor is 21.
24. D **25.** F **26.** D
27. **a.** 2 packages of hot dogs and 3 packages of buns; 1 hot dog and 1 bun
    b. 10 packages of hot dogs and 15 packages of buns; 4 hot dogs and 4 buns
28. 20: each gets 1 cookie and 2 carrot sticks
    10: each gets 2 cookies and 4 carrot sticks
    5: each gets 4 cookies and 8 carrot sticks
    4: each gets 5 cookies and 10 carrot sticks
    2: each gets 10 cookies and 20 carrot sticks
    1: gets it all: 20 cookies and 40 carrot sticks

29. Answers will vary. For example, a problem using common factors might be: "The Morgan family buys a 12-pack of bottled water and a 24-pack of boxes of raisins. Each person in the family gets the same number of bottles of water and the same number of boxes of raisins. How many people might there be in the Morgan family?" The Morgan family could have 1, 2, 3, 4, 6, or 12 people. I know I'm right because these numbers are common factors of 12 and 24. An example of a problem using common multiples is: "John eats an apple once a week. Ruth eats an apple every third day. If they both eat an apple today, when will John and Ruth next eat an apple on the same day?" John and Ruth will both eat apples on the same day in 21 days. I know I'm right because this problem involves overlapping cycles, so it can be solved with common multiples.

30. G

Connections

31. 7 is a factor of 63. 9 is a factor of 63. 7 is a divisor of 63. 9 is a divisor of 63. The product of 7 and 9 is 63. The product of 9 and 7 is 63. 63 is divisible by 7. 63 is divisible by 9. 63 is a multiple of 7. 63 is a multiple of 9.

32. **a.** 4 **b.** 10

33. 4,995,000; add 3 zeros for the three factors of 10.

34. **a.** In 2 hours, the jet will travel $12 \times 2 \times 60 = 1{,}440$ kilometers. In 6 hours, the jet will travel $1{,}440 \times 3 = 4{,}320$ kilometers.

b. In 6 hours, the jet will travel $4{,}320 - 1{,}440 = 2{,}880$ kilometers more than in 2 hours.

c. In 4 hours, he would travel twice as many miles as in 2 hours, or 2,880 kilometers.

35. Students need to be able to reason proportionally (without knowing that vocabulary) to move from 20 minutes in 1 day to 1 hour in 3 days to 12 hours in 36 days. Julio gains 12 hours in 36 days. Since 12 is a factor of 36, both watches will be correct again 36 days after they set the watches, or at 9:00 A.M. on the 6th Tuesday.

36. This question also asks students to reason proportionally.

a. $9 \times 5 \times 7$ will be 3 times as great as $3 \times 5 \times 7$, so 315.

b. $3 \times 5 \times 14$ will be twice as great as $3 \times 5 \times 7$, so 210.

c. $3 \times 50 \times 7$ will be 10 times as great as $3 \times 5 \times 7$, so 1,050.

d. $3 \times 25 \times 7$ will be 5 times as great as $3 \times 5 \times 7$, so 525.

Extensions

37. $3 \times 4 \times 5 + 1 = 61$ (Find the least common multiple of 2, 3, 4, 5, and 6, and add 1.)

38. 14 and 35 **39.** 90 years

40. **a.** 36 **b.** 0

c. Eric forgot that $3 \times 4 = 4 \times 3$. (Multiplication is commutative.)

41. **a.** $1 + 3 + 5 + 7 + 9 = 25$;
$1 + 3 + 5 + 7 + 9 + 11 = 36$;
$1 + 3 + 5 + 7 + 9 + 11 + 13 = 49$;
$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64$

b. $1 + 3 + 5 + \ldots + 39 = 20^2 = 400$

c. Row 24; 47. The sum will be 576 in row 24, because 576 is 24^2. The last number in this row is 47 because 47 is the twenty-fourth odd number. (This famous pattern is the sum of the consecutive odd numbers: the sum in each row is the square of the number of numbers in the row.)

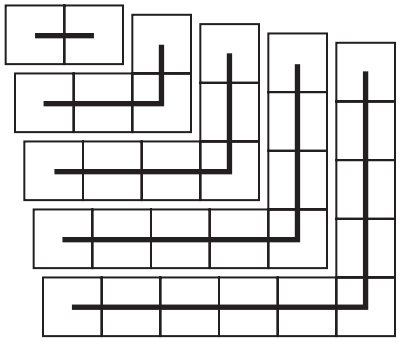
42. **a.** Tiles can be used to set up a visual display of this problem. From the pattern, you can see that adding the first n consecutive even numbers is the same as multiplying n times $(n + 1)$. So, the next four rows are as follows:

$2 + 4 + 6 + 8 + 10 = 30$ (which is 5×6)

$2 + 4 + 6 + 8 + 10 + 12 = 42$
(which is 6×7)

$2 + 4 + 6 + 8 + 10 + 12 + 14 = 56$
(which is 7×8)

$2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 = 72$
(which is 8×9)



b. $2 + 4 + 6 + \ldots + 40 = 420$
(which is 20×21)

c. Row 10 since $10 \times 11 = 110$; 20 because 20 is the tenth even number.

43. a. 12-year cicadas would meet 2-year predators either every time they emerge or never. The 13-year cicadas would encounter predators every other time they emerge, so they could be better or worse off depending on whether the predator came out on odd or even years.

b. The 12-year cicadas will meet both types of predators every time they emerge. The 13-year cicadas will meet the 2-year predators every other time they emerge, and the 3-year predators every third time they emerge. This means that it will be 6 cycles or 78 years before the 13-year locusts have to face both predators again. They are better off than the 12-year cicadas.

44. Yes; there are 212 dates like this. (Figure 2)

Possible Answers to Mathematical Reflections

1. In the first two problems, it was helpful to find common multiples. In the last two problems, it was helpful to find common factors. When the problem involves the repetitions of two or more events and asks questions about when the events will be in sync, then you need to find common multiples. When the problem involves sharing different amounts equally, common factors will help.

2. List the factors for each number and then find the factors that are in both lists. Of these numbers, choose the greatest.

3. List several multiples for each number and then look for the numbers that are in both lists. Of these numbers, choose the least.

Figure 2

Month	Day	Last Two Digits of the Year	Number of Dates
01	01–31	01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 11, 12, 13, . . . , 31	31
02	01–28	02, 04, 06, 08, 10, 12, 14, 16, 18, 20, 22, 24, 26, . . . , 56	28
03	01–31	03, 06, 09, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, . . . , 93	31
04	01–24	04, 08, 12, 16, 20, 24, 28, . . . , 96 Note: $04 \times 25 \neq 00$	24
05	01–19	05, 10, 15, 20, 25, 30, 35 . . . , 95 Note: $05 \times 20 \neq 00$	19
06	01–16	06, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, . . . , 96	16
07	01–14	07, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98	14
08	01–12	08, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96	12
09	01–11	09, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99	11
10	01–09	10, 20, 30, 40, 50, 60, 70, 80, 90 Note: $10 \times 10 \neq 00$	9
11	01–09	11, 22, 33, 44, 55, 66, 77, 88, 99	9
12	01–08	12, 24, 36, 48, 60, 72, 84, 96	8
		Total Number of Dates	212