Investigation 2

ACE Assignment Choices

Differentiated Instruction
Solutions for All Learners

Problem 2.1

Core 1–9, 22–24, 31, 32
Other *Extensions* 33

Problem 2.2

Core 10–16, 25
Other *Extensions* 34–35; unassigned choices from previous problems

Problem 2.3

Core 17–21, 26–30
Other *Extensions* 36–41; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 19 and other ACE exercises, see the CMP *Special Needs Handbook*.

Applications

1. dimensions: 1×24, 2×12, 3×8, 4×6, 6×4, 8×3, 12×2, 24×1
   factor pairs: 1, 24; 2, 12; 3, 8; 4, 6
2. dimensions: 1×32, 2×16, 4×8, 8×4, 16×2, 32×1
   factor pairs: 1, 32; 2, 16; 4, 8
3. dimensions: 1×48, 2×24, 3×16, 4×12, 6×8, 8×6, 12×4, 16×3, 24×2, 48×1
   factor pairs: 1, 48; 2, 24; 3, 16; 4, 12; 6, 8
4. dimensions: 1×45, 3×15, 5×9, 9×5, 15×3, 45×1
   factor pairs: 1, 45; 3, 15; 5, 9
5. dimensions: 1×60, 2×30, 3×20, 4×15, 5×12, 6×10, 10×6, 12×5, 15×4, 20×3, 30×2, 60×1
   factor pairs: 1, 60; 2, 30; 3, 20; 4, 15; 5, 12; 6, 10
6. dimensions: 1×72, 2×36, 3×24, 4×18, 6×12, 8×9, 9×8, 12×6, 18×4, 24×3, 36×2, 72×1
   factor pairs: 1, 72; 2, 36; 3, 24; 4, 18; 6, 12; 8, 9
7. Prime numbers have only two factors: 2, 3, 5, 7, 11, . . .
8. Square numbers have odd numbers of factors: 4, 9, 16, 25, 36, . . .
9. 25. His number must be a square number because it has an odd number of factors. 16 has five factors and 36 has nine factors.
10. An even number minus an even number is even. Students may use tiles for this, and they may show examples: $16 - 4$ is 12. Since an even number plus an even number is even, an even number minus an even number will still be even. They may give answers in terms of factors of two or in terms of "rectangles with height 2." For example, if you subtract one rectangle with height 2 from another rectangle with height 2, what you will have left will still be a rectangle with height 2.

    You can also see this by writing the equation
    EVEN + EVEN = EVEN
    and subtracting an even number from both sides. You are left with
    EVEN = EVEN − EVEN.

11–13. A student might illustrate this in terms of "rectangles with height 2" and "extra squares."

11. An odd number minus an odd number is even. If you have a rectangle with one extra square and you subtract a rectangle with one extra square, you have "subtracted the extra square" and are left with a rectangle with height 2.
12. An even number minus an odd number is odd. If you have a rectangle and you subtract a rectangle with one extra square, you have broken up a pair of squares and are left with another rectangle with an extra square.

13. An odd number minus an even number is odd. If you have a rectangle with one extra square and you subtract a rectangle, you still have an extra square and are left with a rectangle with an extra square.

14. Evens end in 0, 2, 4, 6, or 8, and they are divisible by two. Odds end in 1, 3, 5, 7, or 9, and they are not divisible by 2.

15. If all the numbers are even or if there is an even number of odd numbers, it is even. Otherwise it is odd.

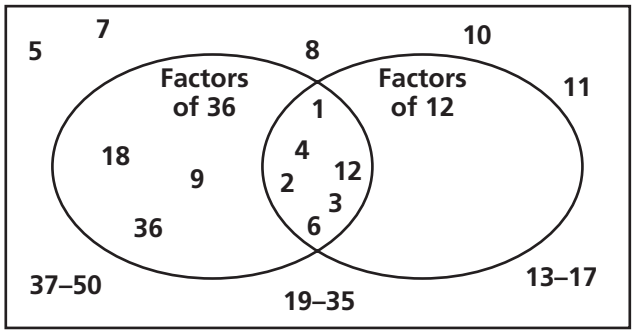
16. a. $2 + 5 \times 3 = 17$

b. $2 \times 5 + 3 = 13$

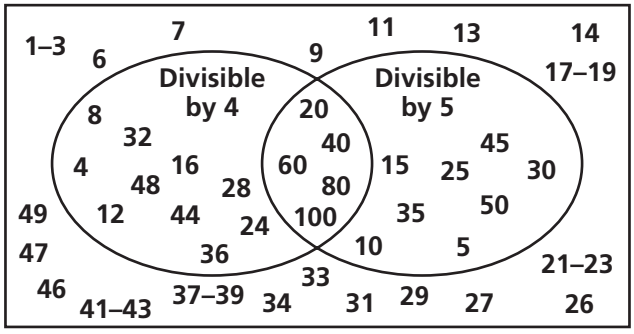
c. $2 \times 5 \times 3 = 30$

d. $2 \times 5 - 3 = 7$

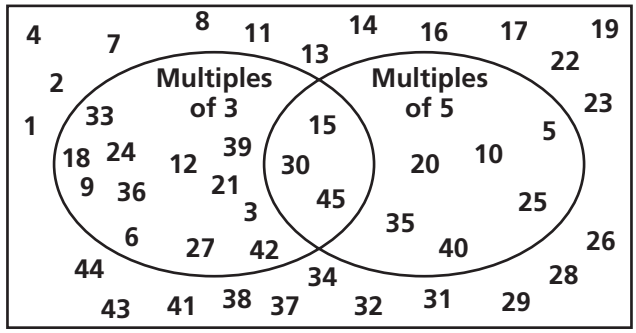
17. There is no number that is a factor of 12 that is not also a factor of 36.



18. Answers may vary. Students should have five of the numbers shown in each region below.



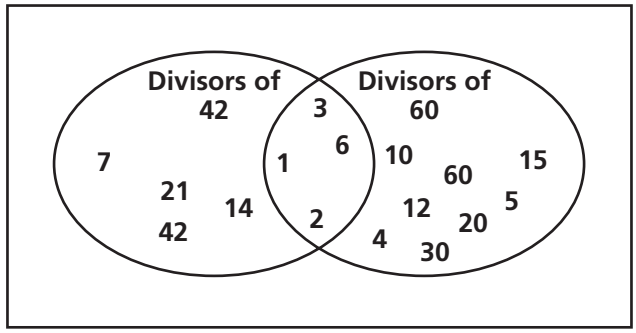
19. a.



b. Possible answers: 2, 4, 7, and 8

c. 15

20. a.



b. 6

21. There are only two: 44 and 88.

22. 100 fans in 1 row, 50 fans in 2 rows, 25 fans in 4 rows, 20 fans in 5 rows, 10 fans in 10 rows, 5 fans in 20 rows, 4 fans in 25 rows, 2 fans in 50 rows, or 1 fan in 100 rows; Answers will vary, for example: I would rather have one long banner that wraps around part of the stadium, so I would choose 100 fans in one row. Or: I would rather have a big square that you could see on TV, so I would choose ten fans in ten rows.

23. 1×64, 2×32, 4×16, 8×8, 16×4, 32×2, and 64×1. Answers will vary as they did in Exercise 22.

24. 2 **25.** A

26. a. 5, 10, 15, 20, 25, 30, 35, 40, 45, 50

b. All of these numbers end in 5 or 0.

c. Those that end in 0.

d. Those divisible by 2 and 5. They end in 0.

e. They are the same—all numbers divisible by 10 are also divisible by 5 and 2.

27. On Thursday, she will earn 16 cents. In 14 days she will earn a total of 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 + 1,024 + 2,048 + 4,096 + 8,192 + 16,384 = 32,766 cents, or $327.66.

28. 500

29. **a.** 8; $6 \times 8 = 48$ **b.** 11; $11 \times 11 = 121$

30. Possible answers: 48 is a multiple of 6; 8 is a divisor of 48; 48 is divisible by 8.

31. H **32.** D

Extensions

33. G

34. Possible answers: $3 \times 10 \times 10$; $2 \times 3 \times 50$

35. **a.** $2 \times 294 = 588$

b. 1, 588; 2, 294; 3, 196; 4, 147; 6, 98; 7, 84; 12, 49; 14, 42; 21, 28

c. There are twice as many rectangles as factor pairs: 1×588, 2×294, 3×196, 4×147, 6×98, 7×84, 12×49, 14×42, 21×28, and the reverses 28×21, 42×14, 49×12, 84×7, 98×6, 147×4, 196×3, 294×2, and 588×1.

36. **a.** Possible answers: Multiples of 12 include 12, 24, 36, 48, and 72. Multiples of 12 AND 20 include 60, 120, 180, 240, and 300. Multiples of 20 include 20, 40, 80, 100, and 140. Multiples of neither include 1, 2, 3, 4, and 5.

b. The multiples in the intersection are all divisible by 60.

37. Either two of the numbers are odd or two are even. If we start with an odd, we will have two odds, and if we start with an even, we will have two evens.

38. Since every third number on the number line is divisible by 3, any three consecutive whole numbers must include one that is divisible by 3.

39. 5 **40.** 6

41. (Figure 3) Students might notice that for square numbers the last factor pair, the turn around point, is the one that uses the square root, though they will not use that vocabulary word. Extending this idea to the non-square numbers, they may say that looking for a square near the number being investigated helps; thus, 66 is close to 8×8, and 7 and 8 are not factors of 66, so there is no point in looking further than 6×11.

Possible Answers to Mathematical Reflections

1. When you make rectangles on grid paper, the dimensions of those rectangles are the factor pairs of the number of squares enclosed. The number of rectangles you make is equal to the number of factors. For example, if you had 24 tiles, the dimensions of the rectangles are: 1×24, 24×1, 2×12, 12×2, 3×8, 8×3, 4×6, and 6×4. There are eight rectangles and eight factors of 24. The dimensions of these rectangles are the factor pairs of 24.
2. A Venn diagram uses circles to show sets of numbers or other objects. A number is inside a circle if it has a particular characteristic (such as being a factor of 30 or being a factor of 36). The overlap of the circles contains numbers that have both of these characteristics (such as being a factor of *both* 30 and 36). The area outside the circles contains numbers with none of these characteristics.
3. Look at the factor pairs that are the closest together, or the ones that form the most squarelike rectangle. You know that you have gone far enough when you check numbers up to this point, because it is the turn around point. After this, the factor that is paired with any greater factor will have already been tested.

Figure 3

Number	16	30	36	40	50	64	66
Last Factor Pair	4 × 4	5 × 6	6 × 6	5 × 8	5 × 10	8 × 8	6 × 11

4. The sum of two even numbers is even because you can combine two rectangles with height 2 to get another rectangle with height 2.

The sum of two odd numbers is even. The tile models for the odd numbers each have an extra square. If you combine the models, you can pair the extra squares to form a rectangle.

The sum of an odd number and an even number, or an even number and an odd number, is odd. If you combine the models, you still have an extra square.

The product of two even numbers is even. If you combine an even number of rectangles, you get another rectangle.

The product of two odd numbers is odd. If you combine an odd number of rectangles with an extra square, you get another rectangle with an extra square because you have an odd number of extra squares.

The product of an odd number and an even number, or an even number and an odd number, is even. If you put together an odd number of even rectangles, you get another rectangle. If you put together an even number of rectangles with extra squares, you get another rectangle.