Investigation 1

ACE Assignment Choices

Differentiated Instruction
Solutions for All Learners

Problem 1.1
Core 1–7
Other *Connections* 28–31, *Extensions* 39

Problem 1.2
Core 8, 10–13, 15, 16
Other *Applications* 9, 14; *Connections* 32–34; *Extensions* 40–44; unassigned choices from previous problems

Problem 1.3
Core 17, 19–21, 23, 24, 27, 35–37
Other *Applications* 18, 22, 25, 26; *Connections* 38; *Extensions* 45–49; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 18 and other ACE exercises, see the CMP *Special Needs Handbook*.

Applications

1. Divide 24 by 12 to see if you get a whole number. Since $12 \times 2 = 24$ or $24 \div 12 = 2$, 12 is a factor.
2. 4 **3.** 9 **4.** 8 **5.** 9
6. Divide 291 by 7 to see if the answer is a whole number. Since $291 \div 7 = 41.571428\ldots$, 7 is *not* a factor of 291.
7. D; 1, 2, 3, and 6 are the factors of 6; 1 and 17 are the factors of 17; 1, 5, and 25 are the factors of 25; 1, 2, 3, 4, 6, 9, 12, 18, and 36 are the factors of 36. So, 36 has the most factors.
8. Check every number beginning with 1 until you begin to get the same factors over again. With each small number factor you find, you will find a second factor when you divide.

   Factors of 110 are: 1, 2, 5, 10, 11, 22, 55, 110. I know I have found all of the factors because I checked all the numbers from 1 to 10, and then started getting repeats.
9. Answers will vary. 30 is the least possibility. Others are all multiples of 30. They may only have the numbers 1 and 30 as additional common factors. Common factors of any two such numbers must include 1, $2 \times 3 = 6$, $2 \times 5 = 10$, $3 \times 5 = 15$ and $2 \times 3 \times 5 = 30$ in addition to 2, 3, and 5.
10. **a.** 6; Since the result is a whole number, 14 is a factor of 84.

    b. 5.6; Since the result is not a whole number, 15 is not a factor of 84.
11. I agree because a factor of a number is a divisor of the number.
12. **a.** Yes, $18 \div 6 = 3$.

    b. No, $6 \div 18 = 0.3333\ldots$.
13. 2, 8, 16
14. No. For example, 25 has only factors 1, 5, 25. There is no other factor relationship among the factors except the trivial cases of $1 \times 5 = 5$ and $1 \times 25 = 25$.
15. **a.** In the Factor Game, your opponent scores points for proper factors of the number you choose. The only proper factor of prime numbers, such as 2, 3, or 7, is 1.

    b. Some numbers, such as 12, 20, and 30, have many proper factors that would give your opponent more points. Some numbers, such as 9, 15, and 25, have fewer factors and would give you more points.
16. The proper factors do not include the number itself.
17. 4 is a factor of 28; 7 is a factor of 28; 4 is a divisor of 28; 7 is a divisor of 28; 28 is a multiple of 4; 28 is a multiple of 7; 28 is the product of 4 and 7; 28 is the product of 7 and 4; 28 is divisible by 4; 28 is divisible by 7.

18. a. Moving the paper clip from 6 to make the products $5 \times 1, 5 \times 2, 5 \times 3, 5 \times 4, 5 \times 5, 5 \times 7, 5 \times 8$, and 5×9; moving the paper clip from the 5 to make the products $6 \times 1, 6 \times 2, 6 \times 4, 6 \times 6, 6 \times 7, 6 \times 8$, and 6×9.

b. Moving the paper clip from 6 to 3, 4 or 9, makes 15, 20 or 45, respectively; moving the paper clip from the 5 to 7 makes 42.

c. Moving the paper clip from 5 to 7 makes 6×7, which is 42.

d. Possible answer: Moving the 5 to 7 to get 42; this blocks the opponent and gets 3 in a row.

19. a. $3 \times 1 = 3; 3 \times 2 = 6; 3 \times 3 = 9; 3 \times 4 = 12; 3 \times 5 = 15; 3 \times 6 = 18; 3 \times 7 = 21; 3 \times 8 = 24; 3 \times 9 = 27.$ So you can get a 3, 6, 9, 12, 15, 18, 21, 24, and 27, which are all multiples of 3.

b. $3 \times 11 = 33; 3 \times 13 = 39; 3 \times 17 = 51; 3 \times 19 = 57; 3 \times 20 = 60$ and many others.

c. There are infinitely many multiples of 3.

20. 2 and 9 or 3 and 6.

21. Answers should include two of the following:

16: 2×8 and 4×4	8: 1×8 and 2×4
12: 2×6 and 3×4	6: 1×6 and 2×3
4: 1×4 and 2×2	

22. F; G contains the multiples of 12, H contains 0 which is not a factor of 12, and J contains the proper factors of 12.

23. Possible answer: the factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30; 30 is even; 30, 60, 90, . . . are multiples of 30; 30 is a composite number; 30 has 8 different factors.

24. a. 36 can be found on the product game by 6×6 or 4×9 and is composite.

b. 5 can be found on the product game by only 1×5 and is prime.

c. 7 can be found on the product game by only 1×7 and is prime.

d. 9 can be found on the product game by 1×9 or 3×3 and is composite.

25. Since the numbers on the game board are multiples of the numbers given as possible factors, you could argue in support of Sal's position.

26. 2

27. The two games are similar in that each focuses on the relationships found in a mathematical statement, such as: $3 \times 7 = 21$. This statement means the same thing as other members of its fact family: $21 \div 7 = 3$ and $21 \div 3 = 7$. The Product Game differs from the Factor Game in what the goal is. In the Factor Game you know the 21 and have to look for the 3 and 7 and all the other factors of 21. In the Product Game, you start with the 3 and 7 and find the 21. So you could argue for similar or not similar.

Connections

28. $25 \times 16 = 400$ boards

29. 5 hours $\times$ 60 minutes $=$ 300 minutes; $300 + 30 = 330$ minutes. $330 \div 12 = 27.5$ so they can play 27 games.

30. B; $27 + 31 + 28 = 86; 144 - 86 = 58; 58 \div 2 = 29$. Carlos read 29 pages on Thursday.

31. Answers will vary.

32. a. Since 24 has many factors, it can be divided into many equal parts. Since 23 is prime, it cannot be subdivided. The only proper factors of 25 are 1 and 5, so it can only be subdivided into 5 groups of 5.

b. Possible answers: 12, 18, 20, 28, 30, 32; these numbers have many factors.

33. a. Because 60 has many factors and 59 and 61 do not.

b. Possible answer: 100, since it has many factors.

34. a. Yes, $132 = 12 \times 11$, so it is divisible by 12. Yes, $132 = 3 \times 44$; yes, $132 = 4 \times 33$.

b. Yes, $160 = 10 \times 16$; yes, $160 = 2 \times 80$; yes, $160 = 5 \times 32$.

c. Yes, $42 = 6 \times 7$; yes, $42 = 3 \times 14$; yes, $42 = 2 \times 21$.

d. Answers will vary. Numbers that end in 2 or 0 are divisible by 2. If a number is divisible by a number, n, it is divisible by divisors or factors of n. Since 132 is divisible by 12, it must also be divisible by the factors of 12, including 3 and 4.

35. $12 \times 7, 42 \times 2, 4 \times 21, 6 \times 14$, or 3×28

36. $5 \times 29 = 145$

37. $30 \times 10, 2 \times 150, 4 \times 75, 5 \times 60, 6 \times 50, 3 \times 100, 15 \times 20$, or 12×25

38. **a.** Various answers: for example, group sizes (1, 2, 4, 5, 6, 3, 2, 2, 3, 2). The goal is to find 10 numbers whose sum is 30.

b. Group sizes (3, 3, 3, 3, 3, 3, 3, 3, 3, 3). If she does not have ten groups, she could have 1 group of 30 students, 2 groups of 15 students each, 5 groups of 6 students each, 6 groups of 5 students each, 10 groups of 3 students each, 15 groups of 2 students each, or 30 groups of 1 student each.

c. In part (a), the sum of the numbers in the ten groups must be 30. In part (b), we are considering *factors* of 30.

Extensions

39. **a.** $1 + 2 + 4 + 5 + 10 + 20 + 25 + 50 = 117$

b. $1 + 3 + 9 + 11 + 33 = 57$

c. 97

40. The numbers that have two even digits (Clue 2) and give a remainder of 4 when divided by 5 (Clue 1) are 24, 44, 64, and 84. Of these numbers, 64 is the only one with digits that add to 10 (Clue 3). The number is 64.

41. **a.**

First Move	Proper Factors	My Score	Opponent's Score
31	1	31	1
32	1, 2, 4, 8, 16	32	31
33	1, 3, 11	33	15
34	1, 2, 17	34	20
35	1, 5, 7	35	13
36	1, 2, 3, 4, 6, 9, 12, 18	36	55
37	1	37	1
38	1, 2, 19	38	22
39	1, 3, 13	39	17
40	1, 2, 4, 5, 8, 10, 20	40	50
41	1	41	1
42	1, 2, 3, 6, 7, 14, 21	42	54
43	1	43	1
44	1, 2, 4, 11, 22	44	40
45	1, 3, 5, 9, 15	45	33
46	1, 2, 23	46	26
47	1	47	1
48	1, 2, 3, 4, 6, 8, 12, 16, 24	48	76
49	1, 7	49	8

b. 31, 37, 41, 43, and 47

42. **a.** $1 + 7 = 8$ **b.** 49

43. 47; it is the greatest prime number.

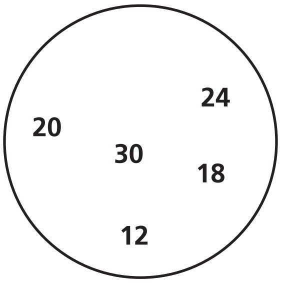
44. 48; the second player gets 76 points—28 more points than the first player.

ACE ANSWERS 1

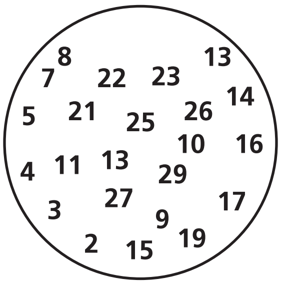
45. 2, 3, and 7; 21 is missing

46. 3, 5, 6 and 7; 25 is missing

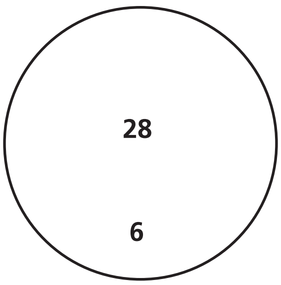
47. a.



Abundant



Deficient



Perfect

b. *Abundant* means "more than enough," which is appropriate since the sum of an abundant number's proper factors is more than the number. *Deficient* means "not enough," which is appropriate since the sum of a deficient number's proper factors is less than the number. *Perfect* means "exactly right," which is appropriate because the sum of a perfect number's proper factors is equal to the number.

c. abundant **d.** deficient

48. 1, 4, 9, 16, 25, 36, 49, 64, 81. If you use 4 dots to represent the number 4, you can make a 2 by 2 square of dots. 9 can be represented by a 3 by 3 square of dots, etc. . . . , so that may be one reason they are called square numbers.

49. a. $1 + 2 + 4 + 8 = 15$; your opponent scores one fewer point.

b. $1 + 2 = 3$; your opponent scores one fewer point.

c. 32, 64, 128, . . . , or any power of 2. The name *near-perfect* fits because the sum of the factors is 1 less than the total needed for the number to be perfect.

Possible Answers to Mathematical Reflections

1. A factor of a number is a number that divides the number with no remainder. (Another way to say this is that a number, a, is a factor of a number, b, if you can find another number, c, that makes a factor pair with a. This means that $a \times c = b$. The existence of the number, c, is the key.)

You find factors of a number by testing each whole number, starting with 1. To test a number you divide it into the target number to see if it divides the target number with no remainder. If it is a divisor of the target number, the quotient is also a factor. You continue to test numbers until you begin to get repeats. Then you have them all.

2. Prime numbers have exactly two factors, themselves and 1.

Composite numbers have more than two factors. The number 1 is neither prime nor composite. It has only 1 factor, so it satisfies neither criterion.

3. The multiples of a number, a, are all the numbers that the original number, a, divides evenly. The least multiple of a number, a, is $a \times 1$ which equals a. This means that every number is a multiple of itself. To find other multiples, you multiply the target number, a, by any whole number.