

## Common Multiples and Common Factors

**M**any things happen over and over again in fixed cycles. For example, a morning news program may have a traffic report every 7 minutes. A train may arrive at a particular station every 12 minutes. A cuckoo clock may sound every 15 minutes.



How can you figure out when two events with different cycles can occur at the same time? Thinking about common factors and common multiples can help you solve such problems.

Let's start by comparing the multiples of 20 and 30.

- The multiples of 20 are 20, 40, 60, 80, 100, 120, 140, 160, 180, . . .
- The multiples of 30 are 30, 60, 90, 120, 150, 180, . . .

The numbers 60, 120, 180, 240, . . . , are multiples of both 20 and 30. We call these numbers **common multiples** of 20 and 30. Of these multiples, 60 is the *least common multiple*.

Now let's compare the factors of 12 and 30.

- The factors of 12 are 1, 2, 3, 4, 6, and 12.
- The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.

The numbers 1, 2, 3, and 6, are factors of both 12 and 30. We call these numbers **common factors** of 12 and 30. Of these factors, 6 is the *greatest common factor*.

## 3.1 Riding Ferris Wheels

One of the more popular rides at a carnival or amusement park is the Ferris wheel.

### Problem 3.1 Choosing Common Multiples or Common Factors

Jeremy and his little sister, Deborah, are at a carnival. There are both a large and a small Ferris wheel. Jeremy gets on the large Ferris wheel at the same time his sister gets on the small Ferris wheel. The rides begin at the same time. For each situation below, decide how many seconds will pass before Jeremy and Deborah are both at the bottom again.

- A. The large wheel makes one revolution in 60 seconds and the small wheel makes one revolution in 20 seconds.
- B. The large wheel makes one revolution in 50 seconds and the small wheel makes one revolution in 30 seconds.
- C. The large wheel makes one revolution in 10 seconds and the small wheel makes one revolution in 7 seconds.
- D. For Questions A–C, determine the number of times each Ferris wheel goes around before Jeremy and his sister are both on the ground again.



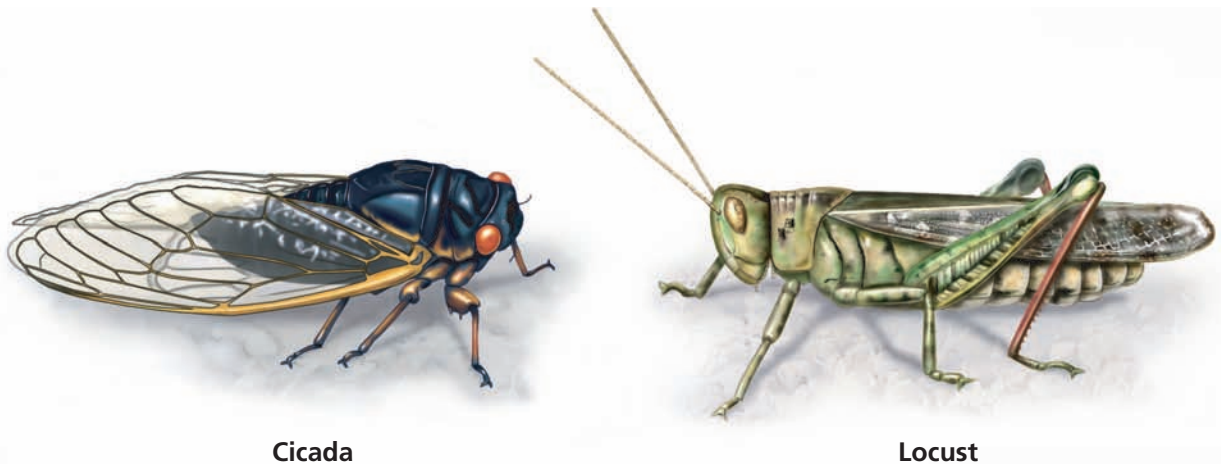
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## 3.2 Looking at Cicada Cycles

**C**icadas (si KAY dahs) spend most of their lives underground. Some populations of cicadas come above ground every 13 years, while others come up every 17 years. Although cicadas do not cause damage directly to fruits and vegetables, they can damage orchards because the female makes slits in trees to lay her eggs.

### Did You Know?

Cicadas are sometimes mistakenly called locusts. A locust is actually a type of grasshopper that looks nothing like a cicada. The error originated when early European settlers in North America encountered large outbreaks of cicadas. The swarms of insects reminded the settlers of stories they knew about swarms of locusts in Egypt.



Female cicadas lay their eggs in tree branches. When the young cicadas hatch, they drop to the ground and burrow into the soil. They remain underground for 13 or 17 years, feeding off juices from tree roots. Several months before they emerge, cicadas tunnel to the surface and wait to come out.

The mass emergence of cicadas is the key to their survival. There may be up to 1.5 million cicadas per acre! Many will be eaten by predators. However, enough will survive to lay eggs, so a new generation can emerge in 13 or 17 years.

**Go Online** For: Information about cicadas  
[PHSchool.com](http://PHSchool.com) Web Code: ame-9031

### Problem 3.2 Choosing Common Multiples or Common Factors

Stephan's grandfather told him about a terrible year when the cicadas were so numerous that they wrecked the buds on all the young trees in his orchard. Stephan conjectured that both 13-year and 17-year cicadas came up that year. Assume that Stephan's conjecture is correct.

- A. How many years after an appearance of 13-year and 17-year cicadas together will both types of cicadas appear together again? Explain.
- B. Suppose there were 12-year, 14-year, and 16-year cicadas, and they all came up this year. How many years will elapse before they all come up together again? Explain.
- C. For Questions A and B, tell whether the answer is less than, greater than, or equal to the product of the cicada cycles.

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## 3.3 Bagging Snacks

You have used common multiples to analyze events that repeat in cycles. Now you will explore problems about sharing items equally. Common factors can help you solve “sharing” problems.

### Problem 3.3 Choosing Common Multiples or Common Factors

Jane and her friends are going on a hiking trip. Jane wants to make snack packs of apples and trail mix to take on the trip. She has 24 apples and 36 small bags of trail mix.

- A.
  1. What is the greatest number of snack packs Jane can make if each pack must have exactly the same number of apples and exactly the same number of bags of trail mix? She doesn't want any apples or trail mix left over. Explain.
  2. Could Jane make a different number of snack packs so that the treats are shared equally? If so, describe each possibility.
  3. Which possibility seems most reasonable to you? Why?

- B.** Suppose that Jane's pet canary has bitten into six of the packages of trail mix and ruined them. Now what is the greatest number of snack packs Jane can make so that the apples and the remaining trail mix are shared equally?

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## 3.4 Planning a Picnic

**M**iriam's uncle runs a small convenience store. He often donates treats for Miriam's school parties.

### Problem 3.4 Choosing Common Multiples or Common Factors

Miriam's uncle donated 120 cans of juice and 90 packs of cheese crackers for the school field trip. Each student is to receive the same number of cans of juice and the same number of packs of crackers.



- A.** What is the greatest number of students that can go on the field trip and share the food equally with no food left over? How many cans of juice and how many packs of crackers will each student receive? Explain.
- B.** Suppose Miriam's uncle eats two packs of crackers before he sends the supplies to school. What is the greatest number of students that can go on the field trip and share the food equally? How many cans of juice and how many packs of crackers will each student receive?

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