Applications



Connections

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Applications

For Exercises 1–6, give the dimensions of each rectangle that can be made from the given number of tiles. Then use the dimensions of the rectangles to list all the factor pairs for each number.

1. 24 **2.** 32 **3.** 48 **4.** 45 **5.** 60 **6.** 72

- 7. What type of number has exactly two factors? Give examples.
- 8. What type of number has an odd number of factors? Give examples.
- **9.** Luke has chosen a mystery number. His number is greater than 12 and less than 40, and it has exactly three factors. What might his number be? Use the display of rectangles for the numbers 1 to 30 from Problem 2.1 to help you find Luke's number. You may also need to think about what the displays for the numbers 31 to 40 would look like.



For Exercises 10–13, make a conjecture about whether each result will be odd or even. Use models, pictures, or other reasoning to support your conjectures.

- **10.** An even number minus an even number
- **11.** An odd number minus an odd number
- **12.** An even number minus an odd number
- **13.** An odd number minus an even number

Homework

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- **14.** How can you tell whether a number is even or odd? Explain or illustrate your answer in at least two ways.
- **15.** How can you determine whether a sum of several numbers, such as 13 + 45 + 24 + 17, is even or odd?
- **16.** Insert operation signs to make the answer correct.

a. 2 5 3 = 17	b. 2 b. 3 = 13
c. 2 5 3 = 30	d. 2 5 3 = 7

17. Copy this Venn diagram and place whole numbers from 1 to 36 in the appropriate regions. Do you notice anything unusual about the diagram?



18. Copy this Venn diagram and find at least five numbers that belong in each region.



- **19. a.** Draw and label a Venn diagram in which one circle represents the multiples of 3 and another circle represents the multiples of 5. Place whole numbers from 1 to 45 in the regions of the diagram.
 - **b.** List four numbers between 1 and 45 that fall in the region outside the circles.
 - **c.** The *common multiples* of 3 and 5 (the numbers that are multiples of both 3 and 5) should be in the intersection of the circles. What is the least common multiple of 3 and 5?
- **20. a.** Draw and label a Venn diagram in which one circle contains the divisors of 42 and another circle contains the divisors of 60.
 - **b.** The *common factors* of 42 and 60 (the numbers that are divisors of both 42 and 60) should be in the intersection of the circles. What is the greatest common factor of 42 and 60?
- **21.** Find all the common multiples of 4 and 11 that are less than 100.

Connections

22. The Olympic photograph below inspired a school pep club to design card displays for football games. Each display uses 100 square cards, At a game, groups of 100 volunteers will hold up the cards to form complete pictures. They are most effective if the volunteers sit in a rectangular arrangements. What rectangular seating arrangements are possible? Which arrangements would you choose? Why?



- **23.** A school band has 64 members. The band marches in the form of a rectangle. What rectangles can the band director make by arranging the members of the band? Which of these arrangements is most appealing to you? Why?
- **24.** How many rectangles can you build with a prime number of square tiles?
- **25. Multiple Choice** What is my number?
 - **Clue 1** My number has two digits, and both digits are even.
 - **Clue 2** The sum of my number's digits is 10.
 - **Clue 3** My number has 4 as a factor.
 - **Clue 4** The difference between the two digits of my number is 6.
 - **A.** 28 **B.** 46 **C.** 64 **D.** 72
- **26. a.** List all the numbers less than or equal to 50 that are divisible by 5.
 - **b.** Describe a pattern you see in your list that you can use to determine whether a large number—such as 1,276,549—is divisible by 5.
 - **c.** Which numbers in your list are divisible by 2?
 - **d.** Which numbers in your list are divisible by 10?
 - **e.** How do the lists in parts (c) and (d) compare? Why does this result make sense?
- **27.** Allie wants to earn some money for a new bike. She tells her dad she will wash the dishes for 2 cents on Monday, for 4 cents on Tuesday, and for 8 cents on Wednesday. If this pattern continued, how much would Allie earn on Thursday? How much would she earn altogether in 14 days?
- **28.** Allie's eccentric aunt, May Belle, hides \$10,000 in \$20 bills under her mattress. If she spends one \$20 bill every day, how many days will it take her to run out of bills?
- **29. a.** What factor is paired with 6 to give 48?**b.** What factor is paired with 11 to give 121?
- **30.** Using the terms *factor*, *divisor*, *multiple*, *product*, and *divisible by*, write as many statements as you can about the number sentence $6 \times 8 = 48$.
- **31.** Multiple Choice Which number is a prime number?
 - **F.** 91 **G.** 51 **H.** 31 **J.** 21

32. Multiple Choice Which number is a composite number?

A. 2 **B.** 79 **C.** 107 **D.** 237



Extensions

- **33. Multiple Choice** Which number is a square number?

 F. 128
 G. 225
 H. 360
 J. 399
- **34.** Find three numbers you can multiply to get 300.
- **35. a.** Below is the complete list of the proper factors of a certain number. What is the number?
 - 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 49, 84, 98, 147, 196, 294
 - **b.** List each of the factor pairs for the number.
 - **c.** How is the list of factor pairs related to the rectangles that could be made to show the number?
- **36. a.** Find at least five numbers that belong in each region of the Venn diagram below.
 - **b.** What do the numbers in the intersection have in common?



Consecutive numbers are whole numbers in a row, such as 31, 32, 33, or 52, 53, 54. Think of different series of consecutive numbers when you work on Exercises 37–40.

37. For any three consecutive numbers, what can you say about odd numbers and even numbers? Explain.

- **38.** Mirari conjectures that, in every three consecutive whole numbers, one number would be divisible by 3. Do you think Mirari is correct? Explain.
- **39.** How many consecutive numbers do you need to guarantee that one of the numbers is divisible by 5?
- **40.** How many consecutive numbers do you need to guarantee that one of the numbers is divisible by 6?
- 41. Jeff is trying to determine when to quit looking for more whole number factors of a number. He has collected data about several numbers. For example, 30 has 1 × 30, 2 × 15, 3 × 10, 5 × 6, and then he can stop looking, because the factor pairs repeat. For 36, he can stop looking when he gets to 6 × 6. For 66, there are no new factor pairs after 6 × 11. Copy and complete the table below. Is there any pattern that would help him know when to stop looking?

Number	16	30	36	40	50	64	66
Last Factor Pair		5 imes 6	6 imes 6				6 × 11



Many conjectures involving whole numbers seem simple, but are actually very difficult to justify. For example, in 1742, a mathematician named Christian Goldbach conjectured that any even number, except 2, could be written as the sum of two prime numbers. For example:

This seems like a pretty simple idea, doesn't it? However, in over 260 years, no one has been able to prove that it is true or find an even number that is not the sum of two prime numbers!

Go PHSchool.com For: Information about Goldbach's Conjecture Web Code: ame-9031