

## Applications

1. Ben claims that 12 is a factor of 24. How can you check to determine whether he is correct?
2. What factor is paired with 6 to give 24?
3. What factor is paired with 5 to give 45?
4. What factor is paired with 3 to give 24?
5. What factor is paired with 6 to give 54?
6. How would you test to see whether 7 is a factor of 291?
7. **Multiple Choice** Which of these numbers has the most factors?  
**A.** 6                      **B.** 17                      **C.** 25                      **D.** 36
8. Lareina understands factors, but sometimes she has trouble finding all the factors of a number. What advice would you give to help her find all the factors of a number? Demonstrate by finding all the factors of 110.
9. Find two numbers that have 2, 3, and 5 as factors. What other factors do the two numbers have in common?
10. **a.** What do you get when you use your calculator to divide 84 by 14? What does this tell you about 14 and 84?  
**b.** What do you get when you use your calculator to divide 84 by 15? What does this tell you about 15 and 84?
11. Ramona says the Factor Game might also be called the Divisor Game. Do you agree? Why or why not?

- 12. a.** Is 6 a divisor of 18? Why or why not?  
**b.** Is 18 a divisor of 6? Why or why not?
- 13.** Which of these numbers are divisors of 64?  
 2 6 8 12 16
- 14.** In Exercise 13, Evan noticed that some of the proper factors of 64 can be multiplied to get another proper factor of 64. For example, 2 and 8 are factors of 64, and 16 is also a factor of 64. Does every number have some factors for which this is true?
- 15. a.** A **prime number** has exactly two factors, 1 and itself. If you circle a prime number in the Factor Game, your opponent will receive at most one point. Explain why. Give some examples.  
**b.** A **composite number** has more than two factors. If you circle a composite number in the Factor Game, your opponent might receive more points than you. Explain why. Give some examples.
- 16.** Why is the set of factors of a number not the same as the set of proper factors of that number?
- 17.** Using the terms *factor*, *divisor*, *multiple*, *product*, and *divisible by*, write as many statements as you can about the number sentence  $4 \times 7 = 28$ .
- 18.** Dewayne and Todd are playing the Product Game. Dewayne's markers are on 16, 18, and 28, and Todd's markers are on 14, 21, and 30. The paper clips are on 5 and 6. It is Dewayne's turn to move a paper clip.
- List the moves Dewayne can make.
  - Which move(s) would give Dewayne three markers in a row?
  - Which move(s) would allow him to block Todd?
  - Which move do you think Dewayne should make? Explain.



For: Help with Exercise 14  
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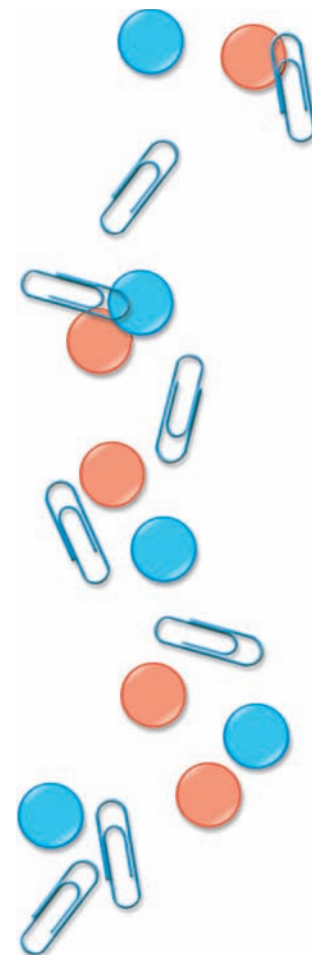
**The Product Game**

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

Factors:

1 2 3 4 5 6 7 8 9

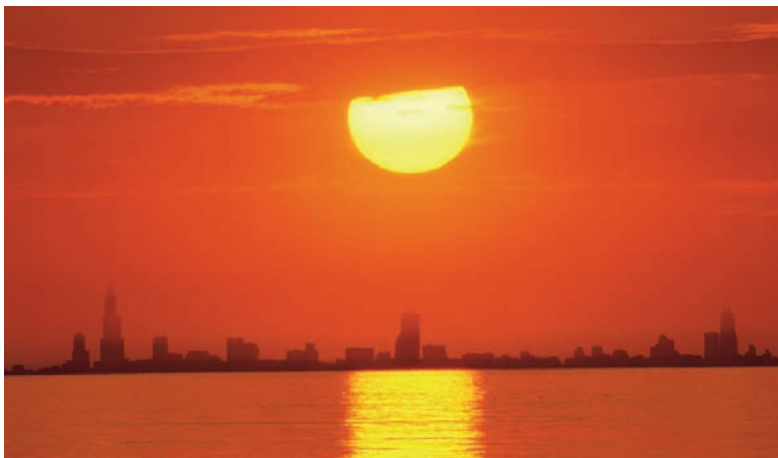
19. **a.** Suppose that one paper clip on the Product Game board is on 3. What products can you make by moving the other paper clip?  
**b.** List five multiples of 3 that are not on the game board.  
**c.** How many multiples of 3 are there?
20. Davis just marked 18 on the Product Game board. On which factors might the paper clips be placed? List all the possibilities.
21. Find two products on the Product Game board, other than 18, that can be made in more than one way. List all the pairs of factors that give each product.
22. **Multiple Choice** Which set represents all the factors of 12?  
**F.** {1, 2, 3, 4, 6, 12}                      **G.** {12, 24, 36, 48, . . . }  
**H.** {0, 1, 2, 3, 4, 6, 12}                **J.** {1, 2, 3, 4, 6}
23. Use the ideas from this investigation to list at least five facts about the number 30.
24. Determine whether each of the following numbers can be made in more than one way in the Product Game. State whether the number is prime or composite.  
**a.** 36                      **b.** 5                      **c.** 7                      **d.** 9
25. Salvador said that the Product Game might also be called the Multiple Game. Do you agree? Why or why not?
26. On the Product Game board, which number is both a prime number and an even number?
27. Jose says the Factor Game and the Product Game are similar because both involve multiplication. Marcus says they are not similar. With whom do you agree and why?



## Connections

28. Twenty-five classes from Martin Luther King Elementary School will play the Factor Game at their math carnival. Each class has 32 students. How many game boards are needed if each pair of students is to play the game once?

- 29.** As part of the carnival, the school will hold a Factor Game marathon. It takes Archie and Kel an average of 12 minutes to finish one game. About how many games will they finish if they play nonstop from 9:00 A.M. to 2:30 P.M.?
- 30. Multiple Choice** This week Carlos read a book for language arts class. He finished the book on Friday. On Monday he read 27 pages; on Tuesday he read 31 pages; and on Wednesday he read 28 pages. On Thursday and Friday he read the same number of pages each day. The book had 144 pages. How many pages did he read on Thursday?
- A.** 28                      **B.** 29                      **C.** 31                      **D.** 58
- 31.** Write a problem like Exercise 30 about a book you have read recently.
- 32.** Long ago, people observed the sun's rising and setting over and over at about equal intervals. They decided to use the amount of time between two sunrises as the length of a day. They divided the day into 24 hours. Use what you know about factors to answer these questions:
- Why is 24 a more convenient choice for the number of hours in a day than 23 or 25?
  - If you could select a number different from 24 to represent the number of hours in a day, what number would you choose? Why?



- 33. a.** In developing the ways in which we calculate time, astronomers divided an hour into 60 minutes. Why is 60 a good choice (better than 59 or 61)?
- b.** If you could select another number to represent the number of minutes in an hour, what would be a good choice? Why?
- 34. a.** Is 132 divisible by 12? By 3? By 4?
- b.** Is 160 divisible by 10? By 2? By 5?
- c.** Is 42 divisible by 6? By 3? By 2?
- d.** What patterns do you see in parts (a), (b), and (c)?

**For Exercises 35–37, find two numbers that can be multiplied to give each product. Do not use 1 as one of the numbers.**

**35.** 84

**36.** 145

**37.** 300

- 38. a.** Ms. Diaz wants to divide her class of 30 students into 10 groups, not necessarily of equal size. What are some of her choices?
- b.** Ms. Diaz wants to divide her class of 30 students into equal-sized groups. What are her choices?
- c.** How is the thinking you did in part (a) different from the thinking you did in part (b)?

## Extensions

- 39.** Jocelyn and Moesha decide to play the Factor Game on a 100-board, which includes the whole numbers from 1 to 100.
- a.** What will Jocelyn's score be if Moesha chooses 100 as her first move?
- b.** What will Jocelyn's score be if Moesha chooses 99 as her first move?
- c.** What is the best first move on a 100-board?
- 40.** What is my number?
- Clue 1** When you divide my number by 5, the remainder is 4.
- Clue 2** My number has two digits, and both digits are even.
- Clue 3** The sum of the digits is 10.

- 41.** The Factor Game can be played on a 49-board, which includes the whole numbers from 1 to 49.
- Use your table for analyzing first moves on a 30-board from Problem 1.2. Extend it to include all the numbers on a 49-board.
  - What new primes do you find?

### The Factor Game

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

- 42.** Lana and Luis are playing the Factor Game on a 49-board. Lana has the first move and chooses 49.
- How many points does Luis score for this round?
  - How many points does Lana score for this round?
- 43.** What is the best first move on a 49-board? Why?
- 44.** What is the worst first move on a 49-board? Why?

- 45.** What three factors were used to make this Product Game board? What product is missing from the grid?

4	6	9
14	?	49

Factors: \_\_\_\_\_

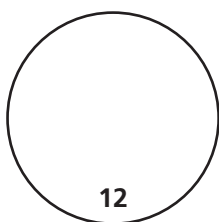
- 46.** What four factors were used to make this Product Game board? What product is missing from the grid?

9	15	18	
21	?	30	35
	36	42	49

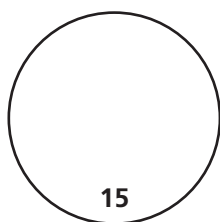
Factors: \_\_\_\_\_

47. The sum of the proper factors of a number may be greater than, less than, or equal to the number. Ancient mathematicians used this idea to classify numbers as *abundant*, *deficient*, and *perfect*. Each whole number greater than 1 falls into one of these three categories.

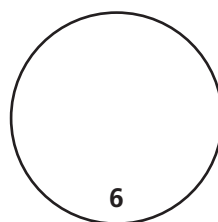
- a. Draw and label three circles as shown below. The numbers 12, 15, and 6 have been placed in the appropriate circles. Use your factor table to determine what each label means. Then, write each whole number from 2 to 30 in the correct circle.



**Abundant**



**Deficient**



**Perfect**

- b. Do the labels seem appropriate? Why or why not?
- c. In which circle would 36 belong?
- d. In which circle would 55 belong?
48. Look at the Product Game board you used in Problem 1.3. Which of the numbers on that board can be formed by placing both paper clips on the same number? These numbers are called *square numbers*. Why do you think they have this name?
49. a. Suppose you choose 16 as a first move in the Factor Game. How many points does your opponent get? How does your opponent's score for this turn compare to yours?
- b. Suppose you choose 4 as a first move. How many points does your opponent get? How does your opponent's score for this turn compare to yours?
- c. Find some other numbers that have the same pattern of scoring as 4 and 16. These numbers could be called *near-perfect numbers*. Why do you think this name fits?

## Did You Know?

Is there a largest perfect number? Mathematicians have been trying for hundreds of years to find the answer to this question. You might like to know that the next largest perfect number after 6 and 28 is 496.

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