# Investigation 3

## ACE Assignment Choices

Differentiated Instruction
Solutions for All Learners

**Problem 3.1**

**Core** 1–3, 36, 37
**Other** *Applications* 4, 5

**Problem 3.2**

**Core** 6–8, 10, 38, 39
**Other** *Applications* 9; unassigned choices from previous problems

**Problem 3.3**

**Core** 11–14, 40–45
**Other** *Applications* 15, *Extensions* 48; unassigned choices from previous problems

**Problem 3.4**

**Core** 18–20, 46, 47
**Other** *Applications* 16, 17; *Extensions* 49; unassigned choices from previous problems

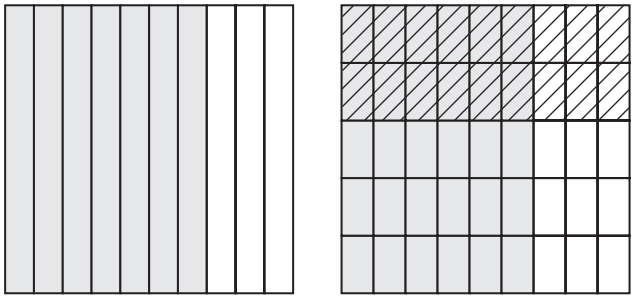
**Problem 3.5**

**Core** 21–33
**Other** *Applications* 34, 35; unassigned choices from previous problems

**Adapted** For suggestions about adapting Exercise 6 and other ACE problems, see the CMP *Special Needs Handbook*.
**Connecting to Prior Units** 37: *Bits and Pieces I*

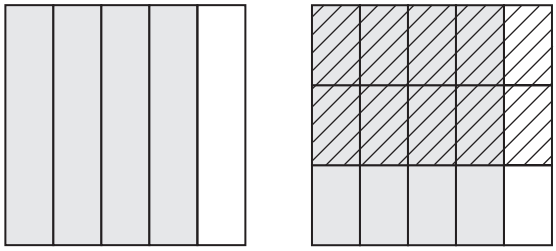
## Applications

**1. a.**



**b.** $\frac{14}{50} = \frac{7}{25}$

**2. a.** $\frac{8}{15}$;



**b.** $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

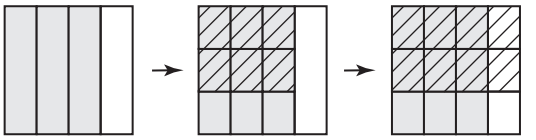
**3. a.** Less than 1; Any number times 1 is itself. $\frac{3}{4}$ times 1 says that you want a $\frac{3}{4}$-part of 1 and $\frac{3}{4}$ of 1 is $\frac{3}{4}$.

**b.** Less than 1; $\frac{2}{3}$ is less than 1 and multiplying $\frac{3}{4}$ by $\frac{2}{3}$ means that you want $\frac{3}{4}$ of, or part of, $\frac{2}{3}$. Part of $\frac{2}{3}$ will be less than $\frac{2}{3}$ and therefore less than 1.

**c.** Less than $\frac{2}{3}$; $\frac{3}{4} \times \frac{2}{3}$ means that you want $\frac{3}{4}$ of, or part of, $\frac{2}{3}$. Part of $\frac{2}{3}$ will be less than $\frac{2}{3}$.

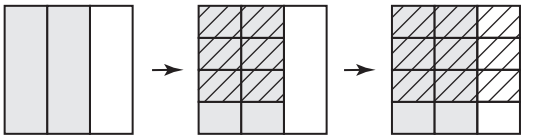
**d.** Less than $\frac{3}{4}$; $\frac{3}{4} \times \frac{2}{3}$ means that you want $\frac{2}{3}$ of, or part of, $\frac{3}{4}$. Part of $\frac{3}{4}$ will be less than $\frac{3}{4}$.

**4. a.** Possible explanation: Finding $\frac{2}{3}$ of $\frac{3}{4}$ means that you have $\frac{3}{4}$ of a pan of brownies and you are going to take $\frac{2}{3}$ of each of the 3 fourths.



**$\frac{3}{4}$ of a pan** **$\frac{2}{3}$ of $\frac{3}{4}$** **$\frac{6}{12}$ or $\frac{1}{2}$ of a pan**

Finding $\frac{3}{4}$ of $\frac{2}{3}$ means that you start with a pan of brownies that is $\frac{2}{3}$ full. You are going to take $\frac{3}{4}$ of each of the 2 thirds.



**$\frac{2}{3}$ of a pan** **$\frac{3}{4}$ of $\frac{2}{3}$** **$\frac{6}{12}$ or $\frac{1}{2}$ of a pan**

**b.** Possible explanation: With either problem you get $\frac{1}{2}$ of a pan of brownies.

**c.** Possible explanation: The answer to both multiplication problems is the same, but how you find that answer is different in each problem when you show it with a brownie pan model. In one you are taking $\frac{2}{3}$ of something, and in the other you are taking $\frac{3}{4}$ of something.

**5. a.** $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$ **b.** $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$

**c.** $\frac{1}{2}$ of $\frac{2}{3} = \frac{2}{6}$, or $\frac{1}{3}$ **d.** $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$

When you take half of a fraction, the denominator doubles. This happens because if you have thirds (e.g. a thirds fraction strip) and you fold each third in half, it makes twice as many pieces because they are half the size of the original third.

**6.** $\frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$ of the class wants to go to Navy Pier.

**7.** $\frac{1}{2}$ of $\frac{7}{8} = \frac{7}{16}$ yard

**8.** Aran: $\frac{1}{2}$

Jon: $\frac{2}{3}$ of $\frac{1}{2} = \frac{2}{6}$, or $\frac{1}{3}$

Kiona: $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$

**9. a.** $\frac{4}{5} \times \frac{3}{4} = \frac{3}{5}$ have brown hair.

**b.** Answers will vary: Any multiple of 20 is possible.

**10. a.** $\frac{2}{9}$ **b.** $\frac{15}{6} = 2\frac{3}{6} = 2\frac{1}{2}$

**c.** $\frac{10}{18} = \frac{5}{9}$ **d.** $\frac{10}{40} = \frac{1}{4}$

**11. a.** Possible answer: almost 3; $\frac{2}{3}$ is a little more than $\frac{1}{2}$. If you had 4 halves, you would get 2, but since it is a little more times 4, you get almost 3.

**b.** Possible answer: about $1\frac{1}{4}$; $\frac{2}{3}$ is between $\frac{1}{2}$ and $\frac{3}{4}$. If you double $\frac{1}{2}$, you get 1, and if you double $\frac{3}{4}$, you get $1\frac{1}{2}$. Doubling $\frac{2}{3}$ will be between 1 and $1\frac{1}{2}$ at about $1\frac{1}{4}$.

**c.** Possible answer: about $1\frac{1}{2}$; if $\frac{2}{3} \times 2$ is about $1\frac{1}{4}$ [see part (b)], then $2\frac{1}{2} \times \frac{2}{3}$ is a little more, or about $1\frac{1}{2}$.

**12. a.** 18 caramel squares

**b.** $\frac{3}{4} \times 2 = 1\frac{1}{2}$ bags will be used.

**13.** $12 \times 11\frac{1}{3} = 136$ tiles

**14.** $11\frac{3}{8} \times 4 + 2 \times 4 = 53\frac{1}{2}$ in.

**15. a.** $\frac{1}{3} \times 18 = 6$ **b.** $\frac{2}{3} \times 18 = 12$

**c.** $\frac{5}{3} \times 18 = 30$ **d.** $1\frac{2}{3} \times 18 = 30$

**16. a.** Possible answers: $\frac{3}{4} \times 4 = 3$ or $\frac{5}{8} \times 8 = 5$

**b.** Possible answers: $\frac{9}{10} \times 1 = \frac{9}{10}$ or $\frac{2}{9} \times 2 = \frac{4}{9}$

**c.** Possible answers: $\frac{5}{6} \times 2 = \frac{10}{6}$ or $\frac{3}{8} \times 5 = \frac{15}{8}$

**d.** Possible answers: $\frac{3}{8} \times 2 = \frac{6}{8}$ or $\frac{1}{15} \times 9 = \frac{9}{15}$

**17. a.** $\frac{3}{4} \times 12 = 9$ cups of pretzels, $\frac{2}{3} \times 12 = 8$ cups of popcorn, $\frac{1}{3}$ of $12 = 4$ cups of peanuts, $\frac{1}{4} \times 12 = 3$ cups chocolate chips

**b.** $\frac{3}{4} \times 15 = \frac{45}{4} = 11\frac{1}{4}$ cups of pretzels,
$\frac{2}{3} \times 15 = \frac{30}{3} = 10$ cups of popcorn,
$\frac{1}{3} \times 15 = 5$ cups of peanuts,
$\frac{1}{4} \times 15 = \frac{15}{4} = 3\frac{3}{4}$ cups of chocolate chips

**18. a.** $\frac{1}{2}$ of the whole sandwich

**b.** $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

**19.** $\frac{3}{4} \times 21 = 15\frac{3}{4}$ **20.** $1\frac{3}{4} \times 2\frac{1}{2} = 4\frac{3}{8}$

**21.** $\frac{20}{36} = \frac{10}{18} = \frac{5}{9}$ **22.** $\frac{14}{56} = \frac{2}{8} = \frac{1}{4}$

**23.** $\frac{203}{27} = 7\frac{14}{27}$ **24.** $\frac{192}{75} = 2\frac{42}{75} = 2\frac{14}{25}$

**25.** $\frac{344}{12} = 28\frac{8}{12} = 28\frac{2}{3}$ **26.** $\frac{36}{56} = \frac{18}{28} = \frac{9}{14}$

**27.** $\frac{99}{60} = 1\frac{39}{60} = 1\frac{13}{20}$ **28.** $\frac{63}{24} = 2\frac{15}{24} = 2\frac{5}{8}$

**29.** $\frac{40}{22} = 1\frac{18}{22} = 1\frac{9}{11}$ **30.** C

**31.** F **32.** B

**33. a.** 60 **b.** 30 **c.** 30 **d.** 6

**e.** 75 **f.** $3\frac{1}{10}$ or 3.1

**34.** $2\frac{13}{32}$ inches

**35. a.** $10\frac{3}{16}$ inches

**b.** No. Since each bead is less than 1 inch, the 16 beads cannot total 16 inches.

## Connections

**36. a.** $3 \times 6 = 18, 6 \times 3 = 18, 18 \div 3 = 6$, and $18 \div 6 = 3$

**b.** $16 \times 3 = 48, 3 \times 16 = 48, 48 \div 3 = 16$, and $48 \div 16 = 3$

**c.** $1\frac{1}{2} \times 7 = 10\frac{1}{2}, 7 \times 1\frac{1}{2} = 10\frac{1}{2}$, $10\frac{1}{2} \div 7 = 1\frac{1}{2}$, and $10\frac{1}{2} \div 1\frac{1}{2} = 7$

**d.** $15 \div 3 = 5, 15 \div 5 = 3, 3 \times 5 = 15$, and $5 \times 3 = 15$

**e.** $100 \div 20 = 5, 100 \div 5 = 20, 5 \times 20 = 100$, and $20 \times 5 = 100$

**f.** $15 \div 1\frac{1}{2} = 10, 15 \div 10 = 1\frac{1}{2}, 1\frac{1}{2} \times 10 = 15$, and $10 \times 1\frac{1}{2} = 15$

**37.** Yes, it is possible. Possible explanation: Suppose Lea had \$4 and Roshaun had \$20. Then Lea would have spent half of \$4, or \$2, and Roshaun would have spent $\frac{1}{4}$ of \$20, or \$5. Thus, Roshaun would have spent more than Lea.

**38.** $\frac{11}{60}$ of the lawn still needs to be mowed.

**39.** $\frac{17}{30}$ bushel of apples

**40.** $6\frac{1}{2}$ **41.** $7\frac{1}{10}$ **42.** $6\frac{19}{30}$ **43.** $2\frac{23}{24}$

**44.** $2\frac{9}{10}$ **45.** $4\frac{1}{15}$

**46. a.** $1\frac{1}{5}$; Fala is fitting together six $\frac{1}{5}$-pie shapes. When she fits them together, five will make one whole and there will be one fifth left. Her approach will lead to the solution $1\frac{1}{5}$.

**b.** $\frac{6}{5}$; If Bri multiplies the numerators and multiplies the denominators, she gets $\frac{6}{5}$.

**c.** 1.2; Hiroshi is using decimals to solve the same problem, so he is most likely to get a decimal answer.

**47.** F

## Extensions

**48. a.** $\frac{1}{4}$ **b.** $\frac{5}{24}$

**49.** B

## Possible Answers to Mathematical Reflections

1. To multiply a fraction by a fraction, you multiply the numerators of each fraction to get the numerator of the product and multiply the denominators of each fraction to get the denominator of the product. If the problem has mixed numbers or whole numbers, change them into fractions. Then multiply using the same method for multiplying a fraction by a fraction.
2. When you multiply a fraction by a fraction, you are taking a part of a part of something. You can use the problem $\frac{1}{4} \times \frac{1}{2}$ and brownie pans to think about this. You have half of a brownie pan, and then you take one-fourth of the half of a pan. This will give you a part of the half, one-eighth, which means the product is less than either of the factors.
3. When you take a fraction of another number, you are taking a part of that number. This implies that you are multiplying. Unlike multiplying with whole numbers where you make some number of whole groups of some amount, you are taking a part of some amount such as half "of" one-fourth.