Investigation 4

Dividing With Fractions

In earlier investigations of this unit, you learned to use addition, subtraction, and multiplication of fractions in a variety of situations. There are times when you also need to divide fractions. To develop ideas about when and how to divide fractions, let's review the meaning of division in problems involving only whole numbers.

Getting Ready for Problem 4.1

Students at Lakeside Middle School raise funds to take a field trip each spring. In each of the following fundraising examples, explain how you recognize what operation(s) to use. Then write a number sentence to show the required calculations.

- The 24 members of the school swim team get dollar-per-mile pledges for a swim marathon they enter. The team goal is to swim 120 miles. How many miles should each swimmer swim?

- There are 360 students going on the field trip. Each school bus carries 30 students. How many buses are needed?
- The school band plans to sell 600 boxes of cookies. There are 20 members in the band. How many boxes should each member sell to reach the goal if each sells the same number of boxes?

Compare your number sentences and reasoning about these problems with classmates. Decide which are correct and why.

4.1 Preparing Food

There are times when the amounts given in a division situation are not whole numbers but fractions. First, you need to understand what division of fractions means. Then you can learn how to calculate quotients when the divisor or the dividend, or both, is a fraction.

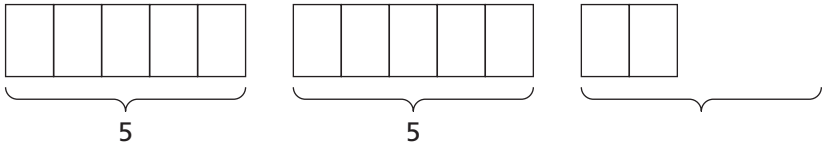
When you do the division $12 \div 5$, what does the answer mean?

The answer should tell you how many fives are in 12 wholes. Because there is not a whole number of fives in 12, you might write:

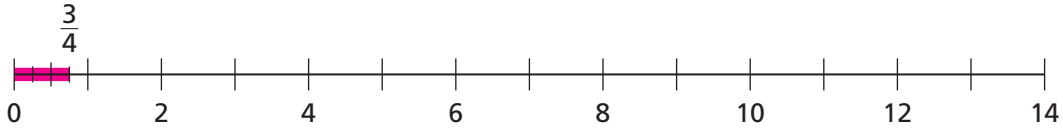
$$12 \div 5 = 2\frac{2}{5}$$

Now the question is, what does the *fractional part* of the answer mean?

The answer means you can make 2 fives and $\frac{2}{5}$ *of another five*.



Suppose you ask, "How many $\frac{3}{4}$'s are in 14?" You can write this as a division problem, $14 \div \frac{3}{4}$.



Can you make a whole number of $\frac{3}{4}$'s out of 14 wholes?

If not, what does the fractional part of the answer mean?

As you work through the problems in this investigation, keep these two questions in mind.

What does the answer to a division problem mean?

What does the fractional part of the answer to a division problem mean?

Problem 4.1 Dividing a Whole Number by a Fraction

Use written explanations or diagrams to show your reasoning for each part. Write a number sentence showing your calculation(s).

A. Naylah plans to make small cheese pizzas to sell at a school fundraiser. She has nine bars of cheese. How many pizzas can she make if each pizza needs the given amount of cheese?

1. $\frac{1}{3}$ bar **2.** $\frac{1}{4}$ bar **3.** $\frac{1}{5}$ bar

4. $\frac{1}{6}$ bar **5.** $\frac{1}{7}$ bar **6.** $\frac{1}{8}$ bar

B. Frank also has nine bars of cheese. How many pizzas can he make if each pizza needs the given amount of cheese?

1. $\frac{1}{3}$ bar **2.** $\frac{2}{3}$ bar **3.** $\frac{3}{3}$ bar **4.** $\frac{4}{3}$ bar

5. The answer to part (2) is a mixed number. What does the fractional part of the answer mean?

C. Use what you learned from Questions A and B to complete the following calculations.

1. $12 \div \frac{1}{3}$ **2.** $12 \div \frac{2}{3}$ **3.** $12 \div \frac{5}{3}$

4. $12 \div \frac{1}{6}$ **5.** $12 \div \frac{5}{6}$ **6.** $12 \div \frac{7}{6}$

7. The answer to part (3) is a mixed number. What does the fractional part of the answer mean in the context of cheese pizzas?

D. 1. Explain why $8 \div \frac{1}{3} = 24$ and $8 \div \frac{2}{3} = 12$.

2. Why is the answer to $8 \div \frac{2}{3}$ exactly half the answer to $8 \div \frac{1}{3}$?

E. Write an algorithm that seems to make sense for dividing any whole number by any fraction.

F. Write a story problem that can be solved using $12 \div \frac{2}{3}$. Explain why the calculation matches the story.

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4.2 Fundraising Continues

While figuring prizes for the games at their fundraiser, students and teachers face more fraction problems!

Problem 4.2 Dividing a Fraction by a Whole Number

Use written explanations or diagrams to show your reasoning for each part. Write a number sentence showing your calculation(s).

A. Ms. Li brings peanuts to be shared equally by members of groups winning each game. How much of a pound of peanuts will each student get in the given situations?

1. Four students share $\frac{1}{2}$ pound of peanuts.

2. Three students share $\frac{1}{4}$ pound of peanuts.

3. Two students share $\frac{1}{5}$ pound of peanuts.

B. A popcorn store donates its different-sized boxes of popcorn for use as prizes at a team competition. How much popcorn does each team member get in the given situations?

1. A two-person team shares a $\frac{3}{4}$-pound box of popcorn equally.

2. A four-person team shares a $\frac{7}{8}$-pound box of popcorn equally.

3. A four-person team shares a $1\frac{1}{2}$-pound box of popcorn equally. (Remember $1\frac{1}{2} = \frac{3}{2}$.)

C. Find each quotient and explain which model you used.

1. $\frac{1}{2} \div 4$

2. $\frac{3}{2} \div 2$

3. $\frac{2}{5} \div 3$

4. $\frac{4}{5} \div 4$

D. What algorithm makes sense for dividing any fraction by any whole number?

E. Write a story problem that can be solved by $\frac{8}{3} \div 4$. Explain why the calculation matches the story.

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4.3 Summer Work

In Problems 4.1 and 4.2, you developed ways of thinking about and solving division problems involving a whole number and a fraction. In the next problem, the questions involve division of a fraction by another fraction.

Problem 4.3 Dividing a Fraction by a Fraction

Rasheed and Ananda have summer jobs at a ribbon company. Answer the questions below. Use written explanations or diagrams in each to show your reasoning. Write a number sentence to show your calculation(s).

A. Rasheed takes a customer order for ribbon badges. It takes $\frac{1}{6}$ yard to make a ribbon for a badge. How many ribbon badges can he make from the given amounts of ribbon? Describe what each fractional part of an answer means.

1. $\frac{1}{2}$ yard

2. $\frac{3}{4}$ yard

3. $2\frac{2}{3}$ yards (Remember $2\frac{2}{3} = \frac{8}{3}$.)

B. Ananda is working on an order for bows. She uses $\frac{2}{3}$ yard of ribbon to make one bow. How many bows can Ananda make from each of the following amounts of ribbon?

1. $\frac{4}{5}$ yard **2.** $1\frac{3}{4}$ yards **3.** $2\frac{1}{3}$ yards

C. Solve each of the following examples as if they were ribbon problems.

1. $\frac{3}{4} \div \frac{2}{3}$ **2.** $1\frac{3}{4} \div \frac{1}{2}$ **3.** $2\frac{3}{4} \div \frac{3}{4}$

D. What algorithm makes sense for dividing any fraction by any fraction?

E. To solve $\frac{3}{4} \div \frac{2}{5}$, Elisha writes, "$\frac{3}{4} \div \frac{2}{5}$ is the same as $\frac{15}{20} \div \frac{8}{20}$. So the answer to $\frac{3}{4} \div \frac{2}{5}$ is the same as $15 \div 8$."

1. Is Elisha's first claim, that $\frac{3}{4} \div \frac{2}{5}$ is the same as $\frac{15}{20} \div \frac{8}{20}$, correct?

2. Is his second claim, that the answer to $\frac{3}{4} \div \frac{2}{5}$ is the same as $15 \div 8$, correct?

3. Use Elisha's method to solve $\frac{3}{5} \div \frac{1}{3}$. Does the method give a correct solution?

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4.4 Writing a Division Algorithm

You are ready now to develop an algorithm for dividing fractions. To get started, you will break division problems into categories and write steps for each kind of problem. Then you can see whether there is one "big" algorithm that will solve them all.

Problem 4.4 Writing a Division Algorithm

A. **1.** Find the quotients in each group below.

Group 1	Group 2	Group 3	Group 4
$\frac{1}{3} \div 9$	$12 \div \frac{1}{6}$	$\frac{5}{6} \div \frac{1}{12}$	$5 \div 1\frac{1}{2}$
$\frac{1}{6} \div 12$	$5 \div \frac{2}{3}$	$\frac{3}{4} \div \frac{3}{4}$	$\frac{1}{2} \div 3\frac{2}{3}$
$\frac{3}{5} \div 6$	$3 \div \frac{2}{5}$	$\frac{9}{5} \div \frac{1}{2}$	$3\frac{1}{3} \div \frac{2}{3}$

2. Describe what the problems in each group have in common.

3. Make up one new problem that fits in each group.

4. Write an algorithm that works for dividing *any* two fractions, including mixed numbers. Test your algorithm on the problems in the table. If necessary, change your algorithm until you think it will work all the time.

B. Use your algorithm to divide.

1. $9 \div \frac{4}{5}$ **2.** $1\frac{7}{8} \div 3$ **3.** $1\frac{2}{3} \div \frac{1}{5}$ **4.** $2\frac{5}{6} \div 1\frac{1}{3}$

C. Here is a multiplication-division fact family for whole numbers:

$5 \times 8 = 40$ $8 \times 5 = 40$ $40 \div 5 = 8$ $40 \div 8 = 5$

1. Complete this multiplication-division fact family for fractions.

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

2. Check the division answers by using your algorithm.

D. For each number sentence, find a value for N that makes the sentence true. If needed, use fact families.

1. $\frac{2}{3} \div \frac{4}{5} = N$ **2.** $\frac{3}{4} \div N = \frac{7}{8}$ **3.** $N \div \frac{1}{4} = 3$

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