Investigation 3

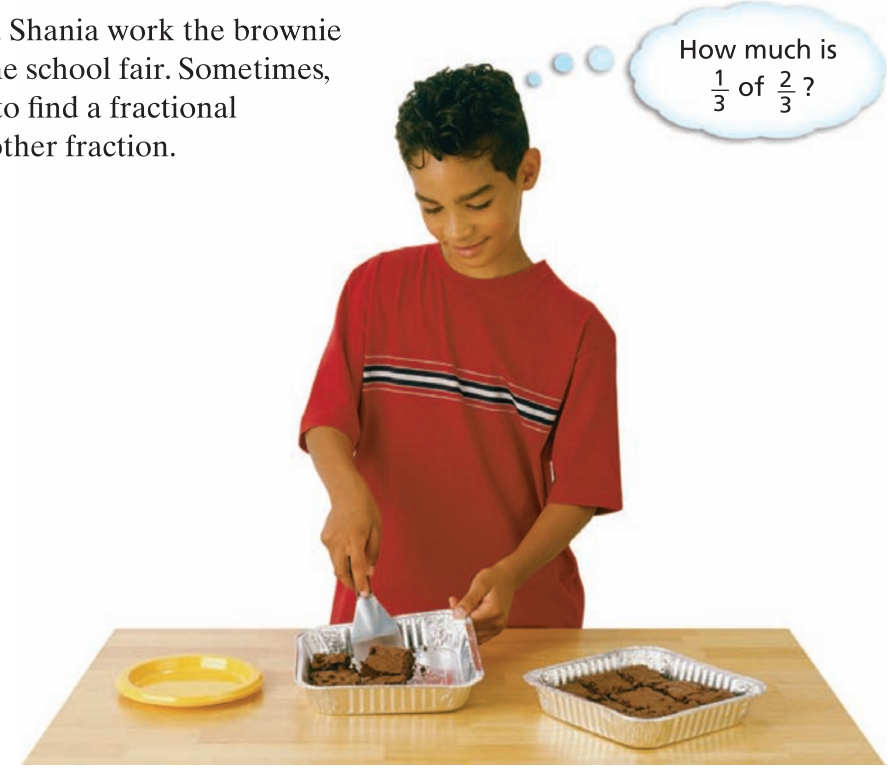
Multiplying With Fractions

Sometimes, instead of adding or subtracting fractions, you need to multiply them. For example, suppose you take inventory at the sporting goods store where you work. There are $13\frac{1}{2}$ boxes of footballs in the stock room, and there are 12 footballs in a full box. How can you find the total number of footballs without opening all the boxes? This situation requires multiplication.

In this investigation, you will relate what you already know about multiplication to situations involving fractions. Remember, to make sense of a situation you can draw a model or change a fraction to an equivalent form. You can also estimate to see if your answer makes sense.

3.1 How Much of the Pan Have We Sold?

Paulo and Shania work the brownie booth at the school fair. Sometimes, they have to find a fractional part of another fraction.

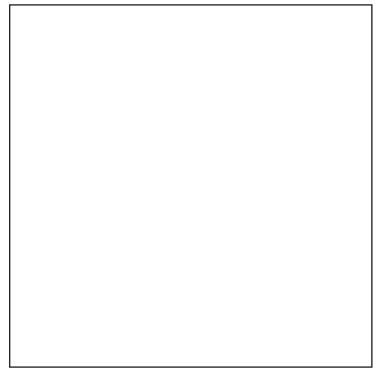


Problem 3.1 A Model for Multiplication

All the pans of brownies are square. A pan of brownies costs \$12. You can buy any fractional part of a pan of brownies and pay that fraction of \$12. For example, $\frac{1}{2}$ of a pan costs $\frac{1}{2}$ of \$12.

A. Mr. Williams asks to buy $\frac{1}{2}$ of a pan that is $\frac{2}{3}$ full.

Model of a Brownie Pan



1. Use a copy of the brownie pan model shown at the right. Draw a picture to show how the brownie pan might look before Mr. Williams buys his brownies.

2. Use a different colored pencil to show the part of the brownies that Mr. Williams buys. Note that Mr. Williams buys *a part of a part* of the brownie pan.

3. What fraction of a whole pan does Mr. Williams buy? What does he pay?

B. Aunt Serena buys $\frac{3}{4}$ of another pan that is half full.

1. Draw a picture to show how the brownie pan might look before Aunt Serena buys her brownies.

2. Use a different colored pencil to show the part of the brownies that Aunt Serena buys.

3. What fraction of a whole pan does Aunt Serena buy? How much did she pay?

C. When mathematicians write $\frac{1}{2}$ of $\frac{1}{4}$, they mean the operation of multiplication, or $\frac{1}{2} \times \frac{1}{4}$. When you multiply a fraction by a fraction, you are finding "a part of a part." Think of each example below as a brownie-pan problem in which you are buying part of a pan that is partly full—a part of a part.

1. $\frac{1}{3} \times \frac{1}{4}$ **2.** $\frac{1}{4} \times \frac{2}{3}$ **3.** $\frac{1}{3} \times \frac{3}{4}$ **4.** $\frac{3}{4} \times \frac{2}{5}$

D. Use estimation to decide if each product is greater than or less than 1. To help, use the "of" interpretation for multiplication. For example, in part (1), think "$\frac{5}{6}$ of $\frac{1}{2}$."

1. $\frac{5}{6} \times \frac{1}{2}$ **2.** $\frac{5}{6} \times 1$ **3.** $\frac{5}{6} \times 2$ **4.** $\frac{3}{7} \times 2$

5. $\frac{3}{4} \times \frac{3}{4}$ **6.** $\frac{1}{2} \times \frac{9}{3}$ **7.** $\frac{1}{2} \times \frac{10}{7}$ **8.** $\frac{9}{10} \times \frac{10}{7}$

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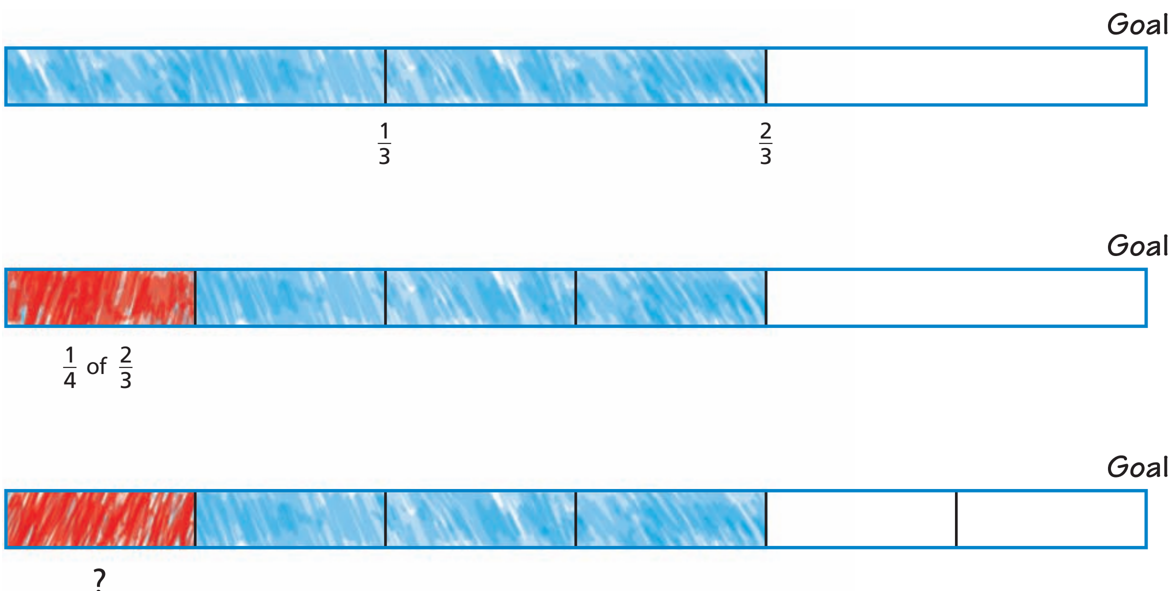
3.2 Finding a Part of a Part

In *Bits and Pieces I*, you used thermometers to show what fraction of a fundraising goal had been met. These thermometers are like number lines. You mark thermometers in the same way you mark number lines to show parts of parts and to name the resulting piece. The fundraising thermometers can help you make sense of the number lines you will use in this problem.

One sixth-grade class raises $\frac{2}{3}$ of their goal in four days. They wonder what fraction of the goal they raise each day on average. To figure this out, they find $\frac{1}{4}$ of $\frac{2}{3}$. One student makes the drawings shown below:



Getting Ready for Problem 3.2

The student above divides the fraction of the goal $\left(\frac{2}{3}\right)$ that is met in four days into fourths to find the length equal to $\frac{1}{4}$ of $\frac{2}{3}$. To figure out the new length, the student divides the whole thermometer into pieces of the same size.

What part of the whole thermometer is $\frac{1}{4}$ of $\frac{2}{3}$?

How would you represent $\frac{1}{4} \times \frac{2}{3}$ on a number line?

How would you represent $\frac{3}{4} \times \frac{2}{3}$ on a number line?

Problem 3.2 Another Model for Multiplication

A. **1.** For parts (a)–(d), use estimation to decide if the product is greater than or less than $\frac{1}{2}$.

a. $\frac{1}{3} \times \frac{1}{2}$ **b.** $\frac{2}{3} \times \frac{1}{2}$ **c.** $\frac{1}{8} \times \frac{4}{5}$ **d.** $\frac{5}{6} \times \frac{3}{4}$

2. Solve parts (a)–(d) above. Use the brownie-pan model or the number-line model.

3. What patterns do you see in your work for parts (a)–(d)?

4. For part (b) above, do each of the following.

a. Write a word problem where it makes sense to use the brownie-pan model to solve the problem.

b. Write a word problem where it makes sense to use the number-line model to solve the problem.

B. Solve the following problems. Write a number sentence for each.

1. Seth runs $\frac{1}{4}$ of a $\frac{1}{2}$-mile relay race. How far does he run?

2. Mali owns $\frac{4}{5}$ of an acre of land. She uses $\frac{1}{3}$ of it for her dog kennel. How much of an acre is used for the kennel?

3. Blaine drives the machine that paints stripes along the highway. He plans to paint a stripe that is $\frac{9}{10}$ of a mile long. He is $\frac{2}{3}$ of the way done when he runs out of paint. How long is the stripe he painted?

C. What observations can you make from Questions A and B that help you write an algorithm for multiplying fractions?

D. Ian says, "When you multiply, the product is greater than each of the two numbers you are multiplying: $3 \times 5 = 15$, and 15 is greater than 3 and 5." Libby disagrees. She says, "When you multiply a fraction by a fraction, the product is less than each of the two fractions you multiplied." Who is correct and why?

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3.3 Modeling More Multiplication Situations

In this problem, you will work with multiplication situations that use fractions, whole numbers, and mixed numbers. It is helpful to estimate first to see if your answer makes sense.

Getting Ready for Problem 3.3

Estimate each product to the nearest whole number (1, 2, 3, . . .).

$\frac{1}{2} \times 2\frac{9}{10}$ $\quad 1\frac{1}{2} \times 2\frac{9}{10}$ $\quad 2\frac{1}{2} \times \frac{4}{7}$ $\quad 3\frac{1}{4} \times 2\frac{11}{12}$

Will the actual product be greater than or less than your whole number estimate?

Problem 3.3 Modeling More Multiplication Situations

For each question:

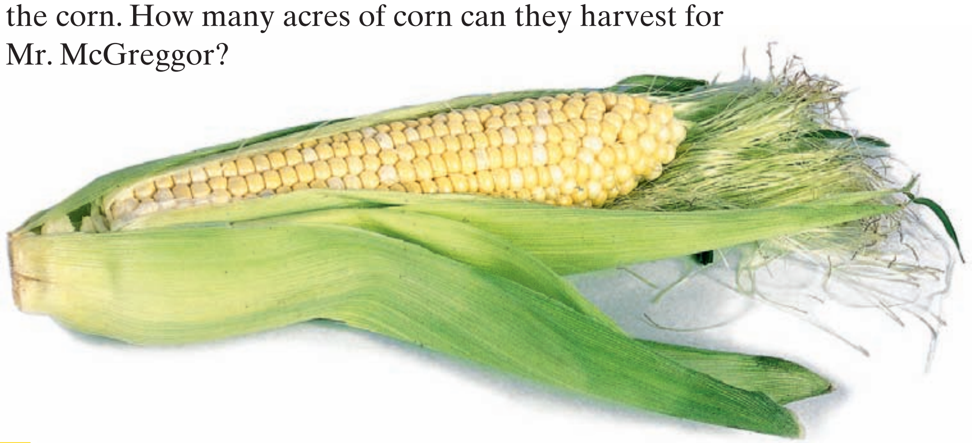
- Estimate the answer.
- Create a model or a diagram to find the exact answer.
- Write a number sentence.

A. The sixth-graders have a fundraiser. They raise enough money to reach $\frac{7}{8}$ of their goal. Nikki raises $\frac{3}{4}$ of this money. What fraction of the goal does Nikki raise?

B. A recipe calls for $\frac{2}{3}$ of a 16-ounce bag of chocolate chips. How many ounces are needed?

C. Mr. Flansburgh buys a $2\frac{1}{2}$-pound wheel of cheese. His family eats $\frac{1}{3}$ of the wheel. How much cheese have they eaten?

D. Peter and Erin run the corn harvester for Mr. McGreggor. They harvest about $2\frac{1}{3}$ acres each day. They have only $10\frac{1}{2}$ days to harvest the corn. How many acres of corn can they harvest for Mr. McGreggor?



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3.4 Changing Forms

You have developed some strategies for modeling multiplication and finding products involving fractions. This problem will give you a chance to further develop your strategies. Before you begin a problem, you should always ask yourself: "About how large will the product be?"

Getting Ready for Problem 3.4

Yuri and Paula are trying to find the following product.

$$2\frac{2}{3} \times \frac{1}{4}$$

Yuri says that if he rewrites $2\frac{2}{3}$, he can use what he knows about multiplying fractions. He writes:

$$\frac{8}{3} \times \frac{1}{4}$$

Paula asks, "Can you do that? Are those two problems the same?"

What do you think about Yuri's idea? Are the two multiplication problems equivalent?

Problem 3.4 Multiplication With Mixed Numbers

A. Use what you know about equivalence and multiplying fractions to first estimate, and then determine, the following products.

1. $2\frac{1}{2} \times 1\frac{1}{6}$ **2.** $3\frac{4}{5} \times \frac{1}{4}$ **3.** $\frac{3}{4} \times 16$

4. $\frac{5}{3} \times 2$ **5.** $1\frac{1}{3} \times 3\frac{6}{7}$ **6.** $\frac{1}{4} \times \frac{9}{4}$

B. Choose two problems from Question A. Draw a picture to prove that your calculations make sense.

C. Takoda answers Question A part (1) by doing the following:

$$\left(2 \times 1\frac{1}{6}\right) + \left(\frac{1}{2} \times 1\frac{1}{6}\right)$$

1. Do you think Takoda's strategy works? Explain.

2. Try Takoda's strategy on parts (2) and (5) in Question A. Does his strategy work? Why or why not?

D. For parts (1)–(3), find a value for N so that the product of $1\frac{1}{2} \times$ N is:

1. between 0 and $1\frac{1}{2}$ **2.** $1\frac{1}{2}$ **3.** between $1\frac{1}{2}$ and 2

4. Describe when a product will be less than each of the two factors.

5. Describe when a product will be greater than each of the two factors.

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3.5 Writing a Multiplication Algorithm

Recall that an algorithm is a reliable mathematical procedure. You have developed algorithms for adding and subtracting fractions. Now you will develop an algorithm for multiplying fractions.

Problem 3.5 Writing a Multiplication Algorithm

A. **1.** Find the products in each group below.

Group 1	Group 2	Group 3
$\frac{1}{3} \times \frac{3}{4}$	$2 \times 1\frac{7}{8}$	$3\frac{2}{3} \times 1\frac{1}{2}$
$\frac{1}{4} \times \frac{2}{5}$	$\frac{2}{5} \times 12$	$2\frac{1}{4} \times 2\frac{5}{6}$
$\frac{2}{3} \times \frac{5}{7}$	$6 \times 1\frac{3}{8}$	$1\frac{1}{5} \times 2\frac{2}{3}$

2. Describe what the problems in each group have in common.

3. Make up one new problem that fits in each group.

4. Write an algorithm that will work for multiplying *any* two fractions, including mixed numbers. Test your algorithm on the problems in the table. If necessary, change your algorithm until you think it will work all the time.

B. Use your algorithm to multiply.

1. $\frac{5}{6} \times \frac{3}{4}$ **2.** $1\frac{2}{3} \times 12$ **3.** $\frac{14}{3} \times \frac{10}{3}$ **4.** $\frac{2}{5} \times 1\frac{1}{2}$

C. Find each product. What pattern do you see? Give another example that fits your pattern.

1. $\frac{7}{8} \times \frac{8}{7}$ **2.** $\frac{1}{9} \times \frac{9}{1}$ **3.** $1\frac{2}{3} \times \frac{3}{5}$ **4.** $11 \times \frac{1}{11}$

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Did You Know?

When you reverse the placement of the numbers in the numerator and the denominator, a new fraction is formed. This new fraction is the **reciprocal** of the original. For example, $\frac{7}{8}$ is the reciprocal of $\frac{8}{7}$, and $\frac{12}{17}$ is the reciprocal of $\frac{17}{12}$, or $1\frac{5}{12}$. Notice that the product of a fraction and its reciprocal is 1.