Investigation 4

ACE Assignment Choices

Differentiated Instruction
Solutions for All Learners

Problem 4.1

Core 1, 2, 26–31

Problem 4.2

Core 3–6, 32
Other unassigned choices from previous problems

Problem 4.3

Core 7–21
Other *Extensions* 34–39; unassigned choices from previous problems

Problem 4.4

Core 22–25, 33
Other *Extensions* 40–44; unassigned choices from previous problems

Adapted For suggestions about adapting Exercises 16–17 and other ACE exercises, see the CMP *Special Needs Handbook*.

Applications

1. a. Answers will vary depending upon how precise the estimate is. Malone is slightly more than $\frac{3}{4}$. Stockton is between $\frac{4}{5}$ and $\frac{7}{8}$.

b. Malone's free-throw record is about 75%. Stockton's free-throw record is about 80% or 85%.

2. Little Neck is more in favor of the proposal. For Little Neck, 41 out of 50 people or more than half responded yes. This is equivalent to having 82 out of 100, or 82% respond yes. There were 100 people from Elmhurst surveyed and only 43, or less than half, responded yes. As a percent, only 43 percent of the people from Elmhurst said yes.

3. A **4.** F **5.** C **6.** F

7. a. $\frac{54}{100}$ of the cats are female.

b. $\frac{46}{100}$ of the cats are male.

c. $\frac{54}{100} = 0.54 = 54\%$; $\frac{46}{100} = 0.46 = 46\%$

8. a. $\frac{17}{100}$ of the cats are kittens.

b. $\frac{83}{100}$ of the cats are adults.

c. $\frac{17}{100} = 0.17$ or 17%; $\frac{83}{100} = 0.83$ or 83%

9. a. $\frac{10}{17}$ of the kittens are male.

b. $\frac{10}{17} =$ about 0.59 or 59%

10. 27% **11.** 20% **12.** 2%

13. 41% **14.** 0% **15.** about 20%

16. Human food only; $\frac{75}{150} = \frac{1}{2}$, 0.5, 50%

17. Pet food only; $\frac{116}{200} = \frac{58}{100} = \frac{29}{50}$, 0.58, 58%

18. Human food only: 50
Pet food only: 30
Human and pet food: 20

19. Human food only: 9
Pet food only: 29
Human and pet food: 12

20. 104%

21. a. 12 students
b. 24 students
c. 18 students

22. a. 0.78

b. $\frac{78}{100}$ or $\frac{39}{50}$

c. 22%. Possible explanation: 100% of the people surveyed represents all the people surveyed, and 78% of those live in a town with a pooper-scooper law, leaving 22%; 0.22, $\frac{22}{100}$ or $\frac{11}{50}$.

d. No. Possible explanation: Percentages are comparisons scaled up or down to be out of 100. 78% means that the number of people who live in a town with a pooper-scooper law compared to the number of people surveyed can be written as 78 out of 100. The actual number surveyed may be out of some other number than 100. For example, it may be that 200 people were surveyed and 156 said yes. The percentage does not tell you how many were surveyed, it only tells you what the amount would be if it were out of 100.

23. a. 34%

b. Possible explanation: 100% represents all the dogs that went to obedience school. 66% is the percent of dogs that passed. The difference between 100% and 66% is the percent of dogs not performing up to par.

24.

Percent	Decimal	Fraction
62%	0.62	$\frac{62}{100} = \frac{31}{50}$
about 44%	about 0.44	$\frac{4}{9}$
123%	1.23	$\frac{123}{100} = 1\frac{23}{50}$
80%	0.8	$\frac{12}{15}$
265%	2.65	$\frac{265}{100} = 2\frac{65}{100}$
55%	0.55	$\frac{55}{100} = \frac{11}{20}$
48%	0.48	$\frac{48}{100} = \frac{12}{25}$
120%	1.20 or 1.2	$\frac{12}{10}$

25. a. 1.05, $\frac{105}{100} = \frac{21}{20}$

b. The number of test questions can be represented by the denominator of any fraction that is equivalent to $\frac{105}{100}$. For example, 20, because $\frac{21}{20} = \frac{105}{100} = 105\%$. 40 will also work, because $\frac{42}{40} = \frac{105}{100} = 105\%$.

Connections

26. $\frac{7}{10} > \frac{5}{8}$

27. $\frac{11}{12} < \frac{12}{13}$

28. $\frac{12}{15} < \frac{12}{14}$

29. $\frac{3}{8} < \frac{4}{8}$

30. $\frac{3}{5} < \frac{4}{6}$

31. $\frac{4}{3} > \frac{15}{12}$

32.

Fraction	Mixed Number
$\frac{13}{5}$	$2\frac{3}{5}$
$\frac{37}{7}$	$5\frac{2}{7}$
$\frac{39}{4}$	$9\frac{3}{4}$
$\frac{23}{3}$	$7\frac{2}{3}$

33. (Figure 3)

Extensions

34. $\frac{1}{2}$, 50%, 0.5

35. $\frac{5}{8}$, 0.625, 62.5%

36. $\frac{11}{60}$, about 0.183, about 18%

37. $\frac{3}{8}$. Possible explanation: If you cut the square into fourths (like a window pane) and then cut each small square in half diagonally, you will get 8 triangular sections just like the shaded ones. Of the 8 triangular sections, 3 are shaded.

38. 0.375. Possible explanation: If you find the area of the white (non-shaded) triangles you can subtract that area from 1 whole to find the area of the shaded triangle. The non-shaded right triangle that sits in the upper right-hand corner takes up $\frac{1}{4}$ or 0.25 of the area of the square. The non-shaded right triangle that sits along the left edge of the square also has an area of 0.25. The non-shaded triangle that sits in the lower right-hand corner of the square has an area of $\frac{1}{8}$ or 0.125 of the square. Altogether, the non-shaded triangles take up 0.625 of the area of the square leaving 0.375 for the shaded triangle.

Figure 3

Percent	10%	$12\frac{1}{2}$%	20%	25%	30%	$33\frac{1}{3}$%	50%	$66\frac{2}{3}$%	75%
Fraction	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{3}{10}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$
Decimal	0.1	0.125	0.2	0.25	0.3	about 0.33	0.5	about 0.67	0.75

39. 50%. Possible explanation: If you draw a horizontal line through the center of the square, you can match up the shaded part on the bottom with the unshaded part on the top, and the unshaded part of the bottom with the shaded part on the top, so about half the square is shaded.

40. 25% discount

41. $112\frac{1}{2}$, $112\frac{1}{2}\%$

42. 275, $137\frac{1}{2}\%$

43. 425, $212\frac{1}{2}\%$

44. **a.** Discount: \$3.60
b. Cost: \$10.50

Possible Answers to Mathematical Reflections

1. "out of 100"

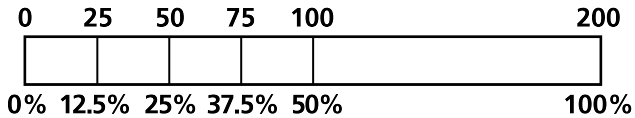
2. **a.** For a whole number percent, you can make the digits of the percent the numerator of the fraction and 100 the denominator of the fraction. To find the decimal, divide 100 into the digits of the percent. For example, $78\% = \frac{78}{100} = 78 \div 100 = 0.78$. For a non-whole number percent, such as 12.5%, this means 12.5 out of 100 or 125 out of 1,000. This would give $\frac{125}{1{,}000}$ or 0.125.

b. When you can change a fraction to an equivalent fraction with 100 as the denominator, the numerator will be the digits of the percent. For example, $\frac{3}{20} = \frac{15}{100} = 15\%$. You can also divide the numerator by the denominator to change the fraction to a decimal. Write the decimal as a percent. For example, $\frac{3}{4} = 3 \div 4 = 0.75 = 75\%$.

c. If the decimal is given to the nearest hundredth, then it is "out of 100". Use the digits in the decimal as the percent. For example, 0.59 = 59 hundredths = 59%. If the decimal extends beyond the hundredths place, as in 0.146, the number formed by the tenths and the hundredths digit is the whole percent. Place the decimal point to the right of the hundredths place so your percent reads 14.6%.

3. Percents are useful when making comparisons because the items being compared are all expressed as "out of 100". Comparing two percents is like comparing two fractions with denominators of 100.

4. To find what percent of 200 is 75 you can scale down the amount to be out of 100. For example, 75 out of 200 is equivalent to 37.5 out of 100, or 37.5%. You can also express 75 out of 200 as a fraction and then divide the numerator by the denominator to find the equivalent decimal and the percent. For example, $\frac{75}{200} = 0.375 = 37.5\%$. Here is one possible way to represent this percent with a percent bar:



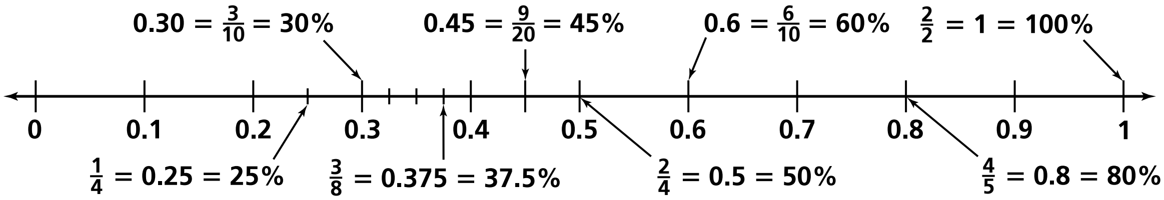
Answer to Looking Back and Looking Ahead

1. **a.** $A = \frac{1}{8}$; $B = \frac{1}{8}$; $C = \frac{3}{16}$; $D = \frac{1}{16}$; $E = \frac{1}{32}$; $F = \frac{5}{32}$; $G = \frac{1}{16}$; $H = \frac{1}{16}$; $I = \frac{1}{8}$; $J = \frac{1}{16}$

b. A = 0.125; B = 0.125; C = 0.1875; D = 0.0625; E = 0.03125; F = 0.15625; G = 0.0625; H = 0.0625; I = 0.125; J = 0.0625

2. Possible solutions provided. Students may have other equivalent forms. (Figure 4)

Figure 4



3. a. $\frac{5}{8} < \frac{7}{8}$. Possible solution: If the fractions have the same denominators, the size of the parts (eighths) are the same. You can compare the numerators or number of parts you have. Since $5 < 7, \frac{5}{8} < \frac{7}{8}$.

b. $\frac{3}{4} > \frac{3}{5}$. Possible solution: If the numerators are the same, each fraction has the same number of parts being used. You can compare the denominators. The smaller the denominator the larger the pieces. Since a fourth is larger than a fifth, $\frac{3}{4} > \frac{3}{5}$.

c. $\frac{3}{4} > \frac{5}{8}$. Possible solution: Change $\frac{3}{4}$ to the equivalent fraction $\frac{6}{8}$ so you are comparing $\frac{6}{8}$ and $\frac{5}{8}$. Since both have the same denominator, compare numerators. Since $6 > 5, \frac{6}{8}$ or $\frac{3}{4} > \frac{5}{8}$.

d. $\frac{3}{8} < \frac{2}{3}$. Possible strategy: These fractions can be compared by comparing each of them to the benchmark $\frac{1}{2}$. Since $\frac{3}{8} < \frac{1}{2}$ and $\frac{2}{3} > \frac{1}{2}$, $\frac{3}{8} < \frac{2}{3}$.

e. $\frac{3}{4} < \frac{4}{5}$. Possible strategy: Both $\frac{3}{4}$ and $\frac{4}{5}$ are one fraction part less than a whole. $\frac{3}{4}$ is $\frac{1}{4}$ less than one whole and $\frac{4}{5}$ is $\frac{1}{5}$ less than one whole. Since $\frac{1}{4}$ is larger than $\frac{1}{5}$, the one part missing from the whole when using fourths is larger than the one part missing from the whole when using fifths. Since $\frac{3}{4}$ has more missing from a whole than $\frac{4}{5}$, then $\frac{3}{4} < \frac{4}{5}$.

f. $\frac{2}{3} > \frac{5}{8}$. Possible strategy: To compare $\frac{2}{3}$ and $\frac{5}{8}$ use equivalent fractions that have the same denominator and then apply the strategy for comparing fractions with the same denominator. Since $\frac{2}{3} = \frac{16}{24}$ and $\frac{5}{8} = \frac{15}{24}$ you can compare $\frac{16}{24}$ and $\frac{15}{24}$. Since $\frac{16}{24} < \frac{15}{24}$, then $\frac{2}{3} > \frac{5}{8}$.

4. Possible strategy: Multiply (or divide) the numerator and denominator of the given fraction by the same number. For example, $\frac{16 \times 2}{20 \times 2} = \frac{32}{40}$ or $\frac{16 \div 2}{20 \div 2} = \frac{8}{10}$.

5. Possible strategy: Some fractions can easily be renamed as equivalent tenths or hundredths (or another decimal place value). For example, $\frac{16 \times 5}{20 \times 5} = \frac{80}{100} = 0.80$ or $\frac{16 \div 2}{20 \div 2} = \frac{8}{10}$ or 0.8. To find the decimal equivalent for any fraction, you can divide the numerator by the denominator. For example, $\frac{16}{20}$ is $16 \div 20 = 0.8$.

6. Possible strategy: To find the percent for a given fraction you can rename the fraction as an equivalent fraction with a denominator of 100. For example, $\frac{16 \times 5}{20 \times 5} = \frac{80}{100}$ or 80%. You can also divide the fraction to find the decimal equivalent. Use the hundredths place in the decimal to help you. For example, $16 \div 20 = 0.8$ or 0.80 which is $\frac{80}{100}$ or 80%.

7. Possible strategy: You can think about equivalent decimals like equivalent fractions. The fraction for 0.18 is $\frac{18}{100}$. Since $\frac{18}{100}$ is equivalent to $\frac{180}{1,000}$, 0.18 is equivalent to 0.180. Each time you place an additional zero to the right of the number, you are renaming the decimal with a different place value and the new decimal is equivalent to the original decimal.

8. Possible strategy: Use a decimal's place value to write the decimal as its fraction equivalent. For example, 0.18 or "eighteen hundredths" can be written as the fraction $\frac{18}{100}$, which is also equivalent to $\frac{9}{50}$ or $\frac{36}{200}$.

9. Possible strategy: To find the percent for a given decimal, use the hundredths place in the decimal to help you. For example, 0.18 or "eighteen hundredths" can be written as the fraction $\frac{18}{100}$ or 18%.

10. Possible strategy: Since percent means "out of 100," you can use the digits in the percent as the numerator of the fraction and 100 as the denominator. Once the percent is written as a fraction you can use it to write the decimal. For example, $35\% = \frac{35}{100}$ or $\frac{7}{20}$.

11. Possible strategy: Since percent means "out of 100," you can think of 3% as 3 out of 100 or "three hundredths". The decimal for three hundredths is 0.03.