# Investigation 🧲

## ACE **Assignment Choices**

Differentiated Instruction

#### **Problem 3.1**

**Core** 1–18 **Other** Connections 51

#### Problem 3.2

**Core** 19-24 Other Connections 52, 53; unassigned choices from previous problems

#### **Problem 3.3**

**Core** 25–28 Other Extensions 55-60; unassigned choices from previous problems

#### Problem 3.4

**Core** 29–32 Other Connections 54; unassigned choices from previous problems

#### **Problem 3.5**

**Core** 33–50 Other unassigned choices from previous problems

Adapted For suggestions about adapting Exercises 1–5 and other ACE exercises, see the CMP Special Needs Handbook.

**Connecting to Prior Units** 52: elementary school measurement; 54: elementary school division concepts

# Applications

In Exercises 1–5, other equivalent fractions are acceptable.

**1.** 
$$\frac{40}{100} = \frac{4}{10} = 0.4$$
  
**2.**  $\frac{32}{100} = 0.32$   
**3.**  $\frac{62.5}{100} = \frac{625}{1000} = 0.625$   
**4.**  $\frac{10}{100} = \frac{1}{10} = 0.1$   
**5.**  $1\frac{6}{100} = \frac{106}{100} = 1.06$ 

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$$1\frac{6}{100} = \frac{106}{100} = 1.06$$

- 6. Possible answers:  $\frac{1}{4}$ ,  $\frac{2}{8}$ ,  $\frac{25}{100}$ ,  $\frac{100}{400}$ ,  $\frac{4}{16}$ . The decimal value 0.25 means  $\frac{25}{100}$ . Since 25 is  $\frac{1}{4}$  of a 100,  $\frac{25}{100}$ is equivalent to  $\frac{1}{4}$ . Each of these fraction amounts will cover  $\frac{1}{4}$  or 25 hundredths of a hundredths grid.
- 7. Possible answers:  $\frac{4}{10}$ ,  $\frac{40}{100}$ ,  $\frac{2}{5}$ ,  $\frac{20}{50}$ . The decimal value 0.40 means  $\frac{40}{100}$ . If you divide 100 into 10 equal parts or tenths, each tenth will have 10 hundredths in it and 4 of those tenths will have 40 hundredths in it, so  $\frac{4}{10}$  is equivalent to  $\frac{40}{100}$ .
- 8. a. 0.50 or 0.5; Possible grid:

**b.** 0.75; Possible grid:

c. 0.99; Possible grid:



#### **d.** 1.30 or 1.3; Possible grid:

_	_	_	_	_	_	_	_	_	_

e. 2.70 or 2.7; Possible grid:

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#### f. 1.60 or 1.6; Possible answer:



- **25. a.** 0.8 or 0.80 (Figure 4) **b.** 0.625 or  $0.62\frac{1}{2}$  (Figure 5)
  - **c.** 0.5 or 0.50 (Figure 6)
- **26.** a. 0.2222 **b.** 1.2222
  - **c.** 0.6666 **d.** 0.6666
  - e. When converting ninths to a decimal, the same number repeats. Some fractions, such as  $\frac{2}{3}$ , have an equivalent form in ninths, so the pattern follows.
- **27.** Possible answers:  $\frac{2}{5} = 0.4$ ;  $\frac{3}{5} = 0.6$ ;  $\frac{4}{5} = 0.8$ . Each fraction is a multiple of  $\frac{1}{5}$ . For example,  $\frac{2}{5}$ is twice as much as  $\frac{1}{5}$ , therefore the decimal for  $\frac{2}{5}$  is twice the decimal for  $\frac{1}{5}$ .
- **b.**  $\frac{1}{2}$  **c.**  $\frac{1}{8}$ **d.**  $\frac{1}{6}$ 28. a.  $\frac{1}{2}$
- **29.** a. Takota's fish; Possible explanation:  $\frac{2}{3}$  is equivalent to  $\frac{16}{24}$ , and  $\frac{5}{8}$  is equivalent to  $\frac{15}{24}$ . Takota's fish is longer because  $\frac{16}{24}$  is greater than  $\frac{15}{24}$ .
  - **b.** Possible answer:  $\frac{2}{3}$  is about 0.67, and  $\frac{5}{8}$  is about 0.63. Since 0.67 and 0.63 are both in hundredths, you can easily compare the numbers. Using decimals may have been easier for them to tell which fish was longer.
- **30.** When Belinda entered  $21 \div 28$  into her calculator, the display read 0.75. This is the decimal equivalent for  $\frac{3}{4}$ . Since  $\frac{21}{28}$  is equivalent to  $\frac{3}{4}$ , the decimal equivalent is also 0.75.

- 32. Possible answer: I would tell the new student to divide the numerator by the denominator on the calculator and round to the nearest hundredth. To show her this makes sense, I would write this decimal as a fraction with 100 in the denominator and show her that the original fraction and this fraction are nearly the same amount. I could do this by finding a number to multiply the numerator and the denominator of the original fraction by that gives a denominator close to 100.
- **33.** 0.205 < 0.21 **34.** 0.1 = 0.1000**35.** 0.04 < 0.050**36.** 1.03 > 0.03**37.**  $\frac{5}{10} < 0.6$ **38.**  $\frac{3}{5} > 0.3$ **39.**  $0.4 = \frac{2}{5}$ **40.**  $0.7 > \frac{1}{2}$ **41.**  $0.52 > \frac{2}{4}$ **42.** 0.41 > 0.405
- 43. Answers may vary.
- 44. Possible answer: 0.9 is greater than 0.45 because 0.9 is equivalent to 0.90 and 90 hundredths is greater than 45 hundredths.
- **45.** Possible answer: 0.75 is greater than 0.60 because it has a larger value in the tenths place.
- **46.** Possible answer: 0.6 and 0.60 are equivalent because they both cover the same area of a hundredths grid.
- **47.** 0.12, 0.127, 0.2, 0.33,  $\frac{45}{10}$
- **48.**  $\frac{3}{1000}$ , 0.005, 0.34,  $\frac{45}{10}$

**49.** 
$$\frac{40}{1000}, \frac{4}{10}, 0.418, 0.48$$

**49.**  $\frac{1}{1000}$ ,  $\frac{1}{10}$ , 0.418, 0.481 **50.** 0.827,  $\frac{987}{1000}$ , 1.23,  $\frac{987}{100}$ 

**31.** C

Figure 4

Figure 5

Figure 6

# Connections

- **51.** a.  $\frac{8}{10}$  or  $\frac{4}{5}$ ; 0.8
  - **b.** Possible answer: If every student were to receive one piece of every pizza, each pizza would have to be divided into ten equal pieces, with each piece being  $\frac{1}{10}$  of a pizza. Each student would receive eight pieces, giving each  $\frac{8}{10}$  of a pizza.



**52.** a. Each part represents  $\frac{1}{10}$  or 0.1 of a centimeter. The following art is not to size.



- **b.** Each part represents  $\frac{1}{100}$  or 0.01 of a centimeter.
- **c.** Each part represents  $\frac{1}{1000}$  or 0.001 of a centimeter.
- 53. a. Yes, it is possible to place 5 numbers between 0.4 and 0.5 by including digits in the place-value spots beyond the tenths place. One possible solution is given. (Figure 7)

- **b.** Yes. One way to place 5 numbers between 0.45 and 0.50 is to include digits in the thousandths place. One possible solution is given. (Figure 8)
- **54.** When you divide you are separating a quantity of something into groups of the same size in the same way, you would share a quantity of something evenly.

### Extensions

**55.** 3. Possible explanation: If you break 12 into fourths or 4 groups of equal size, one group or  $\frac{1}{4}$  of 12 will be 3.



**56.** 6. Possible explanation: If you partition 8 into fourths or 4 groups of equal size, one group or  $\frac{1}{4}$  of 8 is 2. Three of those groups or  $\frac{3}{4}$  of 8 is 6.



**57.** 4. Possible explanation: If you partition 18 into ninths or 9 groups of equal size, one group or  $\frac{1}{9}$  of 18 is 2. Two of those groups or  $\frac{2}{9}$  of 18 is 4.

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**58.**  $\frac{2}{3}$ . Possible explanation: If you partition 3 wholes into 9 parts of equal size, each part will be  $\frac{1}{3}$  of 1 whole and 2 of the nine parts or  $\frac{2}{9}$  of 3 will be 2 of the one-third size parts or  $\frac{2}{3}$  of one of the wholes.



**59.**  $\frac{3}{4}$ . Possible explanation: If you have 3 wholes and partition each whole into fourths, you can take  $\frac{1}{4}$  from each of the 3 wholes. You will have 3 one-fourth-size parts, which is  $\frac{3}{4}$  of one whole.



**60.**  $2\frac{1}{4}$  or  $\frac{9}{4}$  or 2.25. Possible explanation: If you have 3 wholes and partition each whole into fourths, you can take 3 fourth-size pieces or  $\frac{3}{4}$  from each of the 3 wholes. You will have 9 one-fourth-size parts or  $\frac{9}{4}$  which is equivalent to  $2\frac{1}{4}$ .

# Possible Answers to Mathematical Reflections

- One method is to divide the numerator by the denominator. You can check your strategy using a different method. For example, sometimes it is easier to convert the fraction to an equivalent fraction with a denominator of 10, 100, etc. For example, <sup>1</sup>/<sub>5</sub> is equivalent to <sup>2</sup>/<sub>10</sub> or 0.2. At other times, knowing the decimal equivalent for a unit fraction can be a useful method. For example, if you know that <sup>1</sup>/<sub>5</sub> = 0.2, then you can find the decimal equivalent of <sup>2</sup>/<sub>5</sub> by multiplying the decimal for <sup>1</sup>/<sub>5</sub> or 0.2 by 2, the numerator of the fraction <sup>2</sup>/<sub>5</sub>.
- 2. To find a fraction equivalent to a given decimal, use the digits of the decimal as the numerator, and use the place value of the last digit to determine the denominator. For example, with 0.589 the numerator of the fraction is 589. Since the 9 in 0.589 is in the thousandths place, the denominator is 1,000. So 0.589 is equivalent to  $\frac{589}{1,000}$ . You need to be careful with a decimal like 0.005. The numerator of this fraction is 5, not 005. You can ignore leading digits when writing the numerator. Since the 5 in 0.005 is in the thousandths place the fraction for 0.005 is  $\frac{5}{1,000}$ .
- **3.** When comparing 0.57 and 0.559, you first compare the whole numbers. Both numbers do not have a whole number value. Next, compare the place-value spots starting from the tenths and moving to the right one at a time. Both decimals have a 5 in the tenths place indicating that both numbers have 5 tenths in them. When you compare the hundredths place, 0.57 has 7 hundredths and 0.559 has 5 hundredths. Since 0.57 has more hundredths it is the greater number. You do not have to compare any of the other place-value positions because they are less than hundredths. The 9 thousandths in 0.599 is not great enough to make a hundredth, so 0.57 is greater than 0.559.