## Investigation

## ACE

Assignment Choices

## Differentiated Instruction

## Problem 2.1

Core 1-3
Other Connections 46-48

## Problem 2.\&

Core 4-11
Other Extensions 59-64; unassigned choices from previous problems

## Problem 2.3

Core 12-23
Other Extensions 65-69; unassigned choices from previous problems

## Problem 2.4

Core 24
Other Connections 49-58 Extensions 70, 71; unassigned choices from previous problems

## Problem 2.5

Core 25-45
Other Extensions 72; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 2 and other ACE exercises, see the CMP Special Needs Handbook.
Connecting to Prior Units 49 and 51-54: Prime Time

## Applications

1. Each gets $\frac{1}{6}$ bag of popcorn and $\frac{1}{6}$ of each of the three licorice laces or $\frac{1}{2}$ of one lace.
2. Possible answers: The sandwiches can be cut into 9,18 , or 27 pieces.

3. Possible answers: $\frac{8}{24}$ or $\frac{1}{3}$ of the land.
4. Correct
5. Correct
6. Incorrect
7. Incorrect
8. 


9.

10. Possible answer: You could draw a fraction strip and divide it into five equal parts. Shade three of these parts to represent $\frac{3}{5}$. Then divide each of the five parts into two equal parts. You would then have ten equal parts, and six of the parts would be shaded. Therefore, $\frac{3}{5}$ is the same as $\frac{6}{10}$, so $\frac{6}{10}$ is equivalent to $\frac{3}{5}$.
11. $\frac{3}{4}, \frac{6}{8}, \frac{12}{16}$
12. $\frac{8}{10}>\frac{3}{8}$
13. $\frac{2}{3}>\frac{4}{9}$
14. $\frac{3}{5}>\frac{5}{12}$
15. $\frac{1}{3}<\frac{2}{3}$
16. $\frac{3}{4}>\frac{3}{5}$
17. $\frac{3}{2}>\frac{7}{6}$
18. $\frac{8}{12}=\frac{6}{9}$
19. $\frac{9}{10}<\frac{10}{11}$
20. $\frac{3}{12}<\frac{7}{12}$
21. $\frac{5}{6}>\frac{5}{8}$
22. $\frac{3}{7}=\frac{6}{14}$
23. $\frac{4}{5}<\frac{7}{8}$
24. Possible answers:
a. $\frac{4}{15}$
b. $\frac{3}{24}$
c. $\frac{3}{12}$
d. $\frac{7}{20}$
25. between $\frac{1}{2}$ and 1 ; nearer benchmark: $\frac{1}{2}$
26. between 1 and $1 \frac{1}{2}$; nearer benchmark: $1 \frac{1}{2}$
27. between 1 and $1 \frac{1}{2}$; nearer benchmark: 1
28. between 0 and $\frac{1}{2}$; nearer benchmark: 0
29. between $1 \frac{1}{2}$ and 2 ; nearer benchmark: 2
30. between 1 and $1 \frac{1}{2}$; nearer benchmark: 1
31. equal to $\frac{1}{2}$; nearer benchmark: $\frac{1}{2}$
32. equal to $1 \frac{1}{2}$; nearer benchmark: $1 \frac{1}{2}$;
33. between $1 \frac{1}{2}$ and 2 ; nearer benchmark: 2
34. $\frac{7}{3}$ and $2 \frac{1}{3}$ are equivalent. It takes 3 thirds to make 1 whole, so 7 thirds is equivalent to 2 wholes and one third.
35. A
36. G
37. (Figure 7); $\frac{9}{10}=\frac{18}{20}=\frac{27}{30} ; \frac{11}{10}=\frac{22}{20}=\frac{33}{30}$; $2 \frac{3}{10}=2 \frac{6}{20}=2 \frac{9}{30} ; 2 \frac{5}{10}=2 \frac{10}{20}=2 \frac{15}{30}$
38. (Figure 8)
39. Possible answer:

40. (Figure 9)
41.

| Fraction | Mixed Number |
| :---: | :---: |
| $\frac{5}{3}$ | $1 \frac{2}{3}$ |
| $\frac{14}{5}$ | $2 \frac{4}{5}$ |
| $\frac{66}{7}$ | $9 \frac{3}{7}$ |
| $\frac{19}{6}$ | $3 \frac{1}{6}$ |
| $\frac{37}{4}$ | $9 \frac{1}{4}$ |
| $\frac{20}{3}$ | $6 \frac{2}{3}$ |

42. $3 \frac{1}{3}$ miles
43. Both are right because $\frac{28}{4}$ is equivalent to 7 .
44. a. $\frac{5}{3}$
b. $\frac{27}{4}$
c. $\frac{88}{9}$
d. $\frac{30}{7}$
45. a. $5 \frac{2}{4}$ or $5 \frac{1}{2}$
b. $1 \frac{4}{6}$ or $1 \frac{2}{3}$
c. $3 \frac{2}{5}$
d. $4 \frac{4}{8}$ or $4 \frac{1}{2}$

Figure 7


Figure 8


Figure 9


## Connections

46. about $\frac{1}{7}$
47. about $\frac{5}{7}$
48. a.

300 km


180 km


200 km

b. $100 \mathrm{~km}, 60 \mathrm{~km}$, about 67 km . Explanations will vary.
49. C
50. $\frac{20}{60}, \frac{15}{60}, \frac{12}{60}, \frac{10}{60}, \frac{6}{60}, \frac{4}{60}$
51. 12
52. 3
53. 24
54. 9
55. $\frac{1}{4}$
56. $\frac{2}{3}$
57. $\frac{1}{3}$
58. $\frac{2}{5}$

## Extensions

59. (Figure 10)
60. (Figure 11)
61. (Figure 12)
62. (Figure 13)
63. (Figure 14)
64. (Figure 15)
65. $\frac{1}{5}, \frac{2}{10}, \frac{4}{20}, \frac{5}{25}, \frac{6}{30}, \frac{7}{35}, \frac{8}{40}, \frac{9}{45}$
66. $\frac{16}{6}, \frac{24}{9}, \frac{32}{12}, \frac{40}{15}, \frac{48}{18}, \frac{56}{21}, \frac{64}{24}, \frac{72}{27}, \frac{80}{30}, \frac{88}{33}, \frac{96}{36}$,
$\frac{104}{39}, \frac{112}{42}, \frac{120}{45}, \frac{128}{48}$
67. $\frac{5}{3}, \frac{15}{9}, \frac{20}{12}, \frac{25}{15}, \frac{30}{18}, \frac{35}{21}, \frac{40}{24}, \frac{45}{27}, \frac{50}{30}, \frac{55}{33}, \frac{60}{36}, \frac{65}{39}, \frac{70}{42}, \frac{75}{45}, \frac{80}{48}$
68. a. $\frac{3}{4}$
b. $\frac{3}{12}$ or $\frac{1}{4}$
c. $\frac{6}{12}$ or $\frac{1}{2}$
69. a. Possible answers:

$$
\begin{aligned}
& \frac{7}{8}=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8} \\
& \frac{7}{8}=\frac{1}{4}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8} \\
& \frac{7}{8}=\frac{1}{4}+\frac{1}{8}+\frac{1}{2}
\end{aligned}
$$

b. Possible answers:

$$
\begin{aligned}
& \frac{7}{12}=\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12} \\
& \frac{7}{12}=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{12} \\
& \frac{7}{12}=\frac{1}{3}+\frac{1}{6}+\frac{1}{12}
\end{aligned}
$$

70. Possible answers: $\frac{9}{10}, \frac{12}{12}, \frac{12}{11}, \frac{99}{100}, \frac{70}{65}$

Figure 10


Figure 11


Figure 12


Figure 13


Figure 14


Figure 15

71. $\frac{4}{5}$; To compare $\frac{4}{5}$ to $\frac{17}{23}$, you can write equivalent fractions with $5 \times 23$, or 115 , as the denominator. This gives $\frac{4}{5}=\frac{92}{115}$ and $\frac{17}{23}=\frac{85}{115}$. So $\frac{4}{5}$ is larger. To compare $\frac{4}{5}$ and $\frac{51}{68}$, you can use $5 \times 68$, or 340 , as the common denominator to get $\frac{4}{5}=\frac{272}{340}$ and $\frac{51}{68}=\frac{255}{340}$. So $\frac{4}{5}$ is larger.
72. (Figure 16)

## Possible Answers to Mathematical Reflections

1. Equivalent fractions with larger denominators can be found by multiplying the numerator and denominator by the same number. Equivalent fractions with smaller denominators can be found by dividing the numerator and denominator by the same number.
2. To decide if a fraction is between 0 and $\frac{1}{2}$ or between $\frac{1}{2}$ and 1 compare the numerator and the denominator. If the numerator is more than half of the denominator the fraction is between $\frac{1}{2}$ and 1 and if the numerator is less than half of the denominator, the fraction is between 0 and $\frac{1}{2}$.
3. If the denominators are the same, the fraction with the greater numerator is greater. If the numerators are the same, the fraction with the smaller denominator is greater. If the denominators are different, check to see if one fraction is between 0 and $\frac{1}{2}$ and the other is between $\frac{1}{2}$ and 1 . If this is true, the fraction between $\frac{1}{2}$ and 1 is greater. If both the
numerators and the denominators are different, try to change one of the fractions to an equivalent fraction with the same denominator as the other fraction or find an equivalent fraction for each fraction so that the denominators are the same. Then, compare the numerators.
4. To write a mixed number as a fraction take the whole number in the mixed number and multiply it by the denominator of the fraction in the mixed number. To this answer, add the numerator of the fraction. This sum will give you the numerator in the new fraction. Use the new numerator and the given denominator to write the new fraction that is greater than one. For example, in $4 \frac{3}{5}$, you would multiply the 4 by 5 to get 20 or the total number of fifths in the whole number 4. Next, add the 3 fifths in the fraction to the 20 fifths in the whole number, $20+3$, to get the total number of fifths. The result, 23 , becomes the numerator, and the 5 becomes the denominator in $\frac{23}{5}$.
5. To write a fraction greater than one as a mixed number, take the numerator and divide it by the denominator to see how many wholes can be made. Write any remaining parts as a fraction using the original denominator. For example, in $\frac{11}{3}$, the denominator (3) tells us that it takes three thirds to make one whole. Since there are three sets of three in eleven, $\frac{11}{3}$ has 3 wholes with 2 leftover thirds or $3 \frac{2}{3}$.

Figure 16


